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Joint Research Centre

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Step 5: Weighting methods (I)

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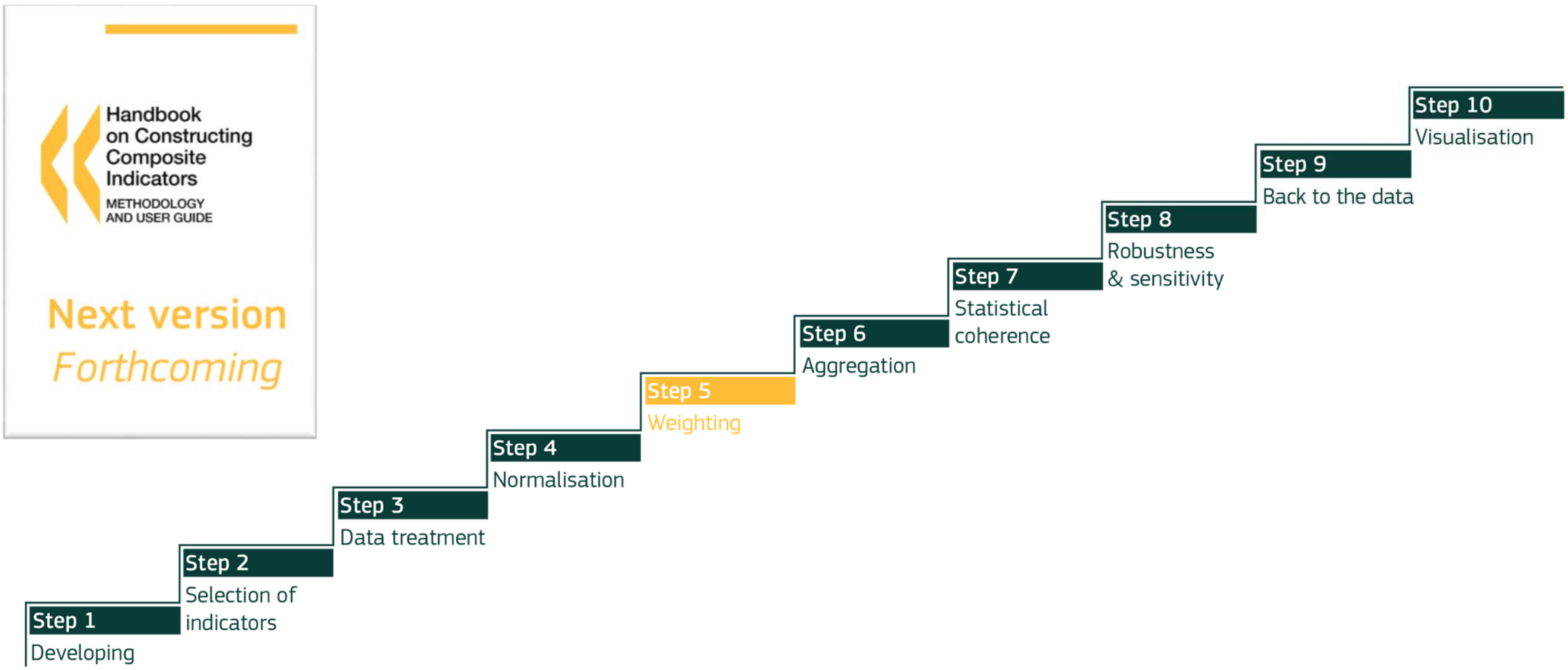
Ten steps



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Weighting

Selecting a weighting scheme for a composite indicator is a delicate task!

- No established methodology on how to choose the best method
- Different stakeholders have different opinions on choosing weighting scheme
- Weights have a strong impact on the final composite indicator score and on the resulting ranking

✓ Take into account the theoretical framework ✓ Be explicit and transparent

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Weighting methods

- **Equal weights**

- **Weights based on statistical methods**
  - Principal component analysis
  - Factor analysis
  - Data envelopment analysis
  - Regression approach

- **Weights based on expert / public opinion**
  - Budget allocation process
  - Analytic hierarchy process
  - Conjoint analysis

Equal Weights

Straightforward method

Easy to communicate

There is no clear reference in the literature about the importance of the elements of the composite indicator

However:

Equal weighing does not guarantee equal importance and equal contribution of the indicators to the composite indicator!

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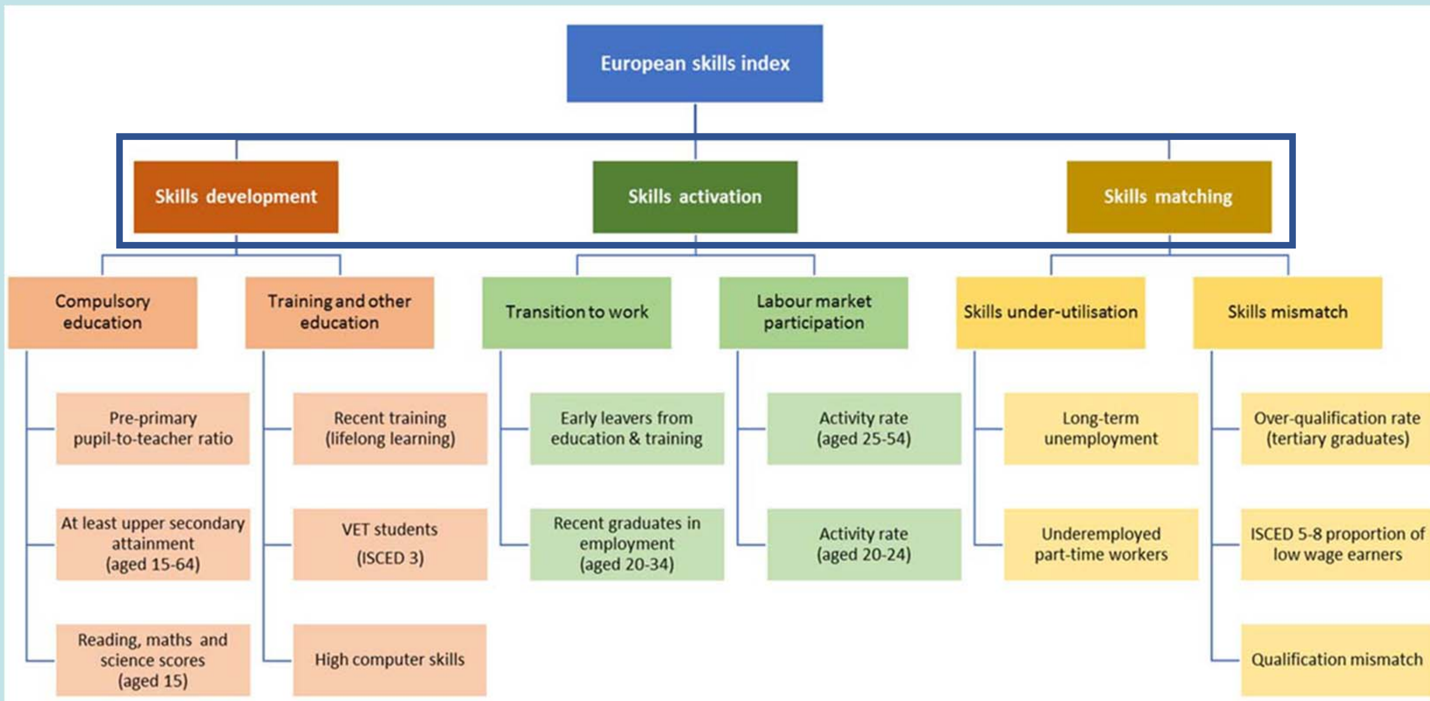
Example 1: European Skills Index



Source: European Skills Index (2018), Cedefop.

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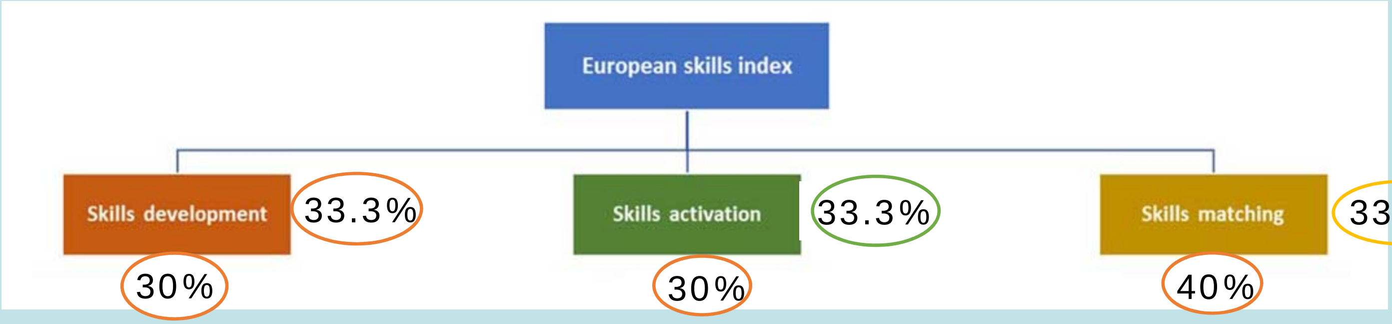
Example: European Skills Index



Equal Weights

Pillars	Pearson Correlation Coefficient	R^2
Skills Development	0.80	0.64
Skills Activation	0.81	0.66
Skills Matching	0.64	0.41

Adjusted Weights

Pillars	Pearson Correlation Coefficient	R^2
Skills Development	0.77	0.59
Skills Activation	0.76	0.58
Skills Matching	0.71	0.50

The unequal weights result in equal importance

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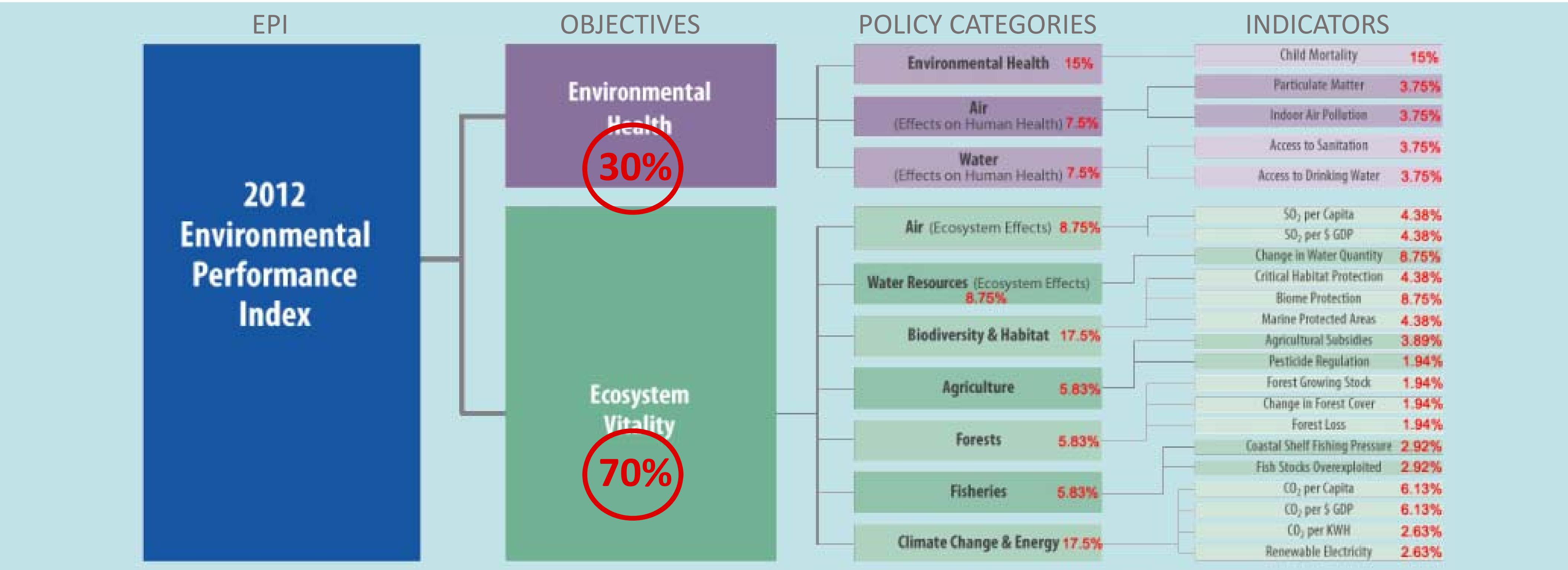
Example 2: Environmental Performance Index



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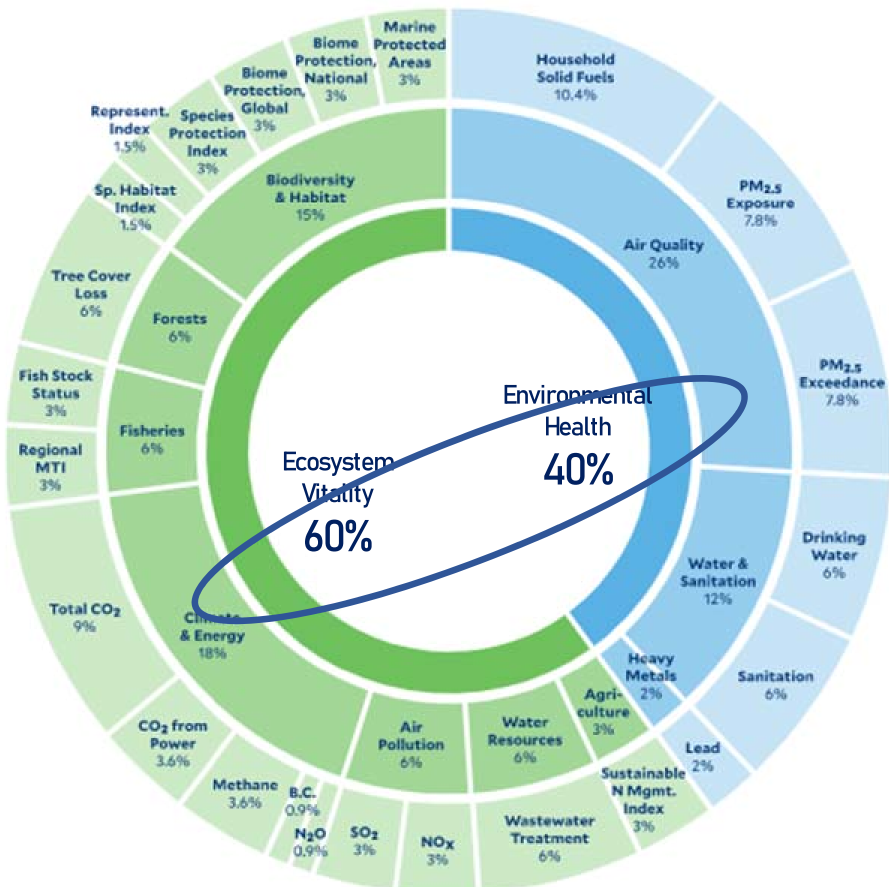
Environmental Performance Index 2018



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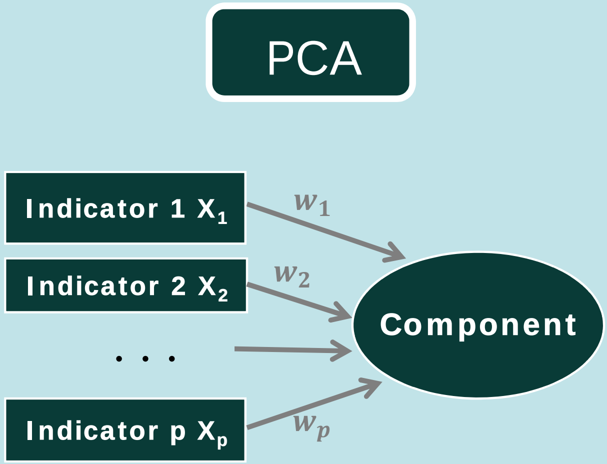
Principal Component Analysis



Observed indicators are reduced into components

Identify a small number of "averages" (PCs) that explain most of the variance observed.

PCA summarizes information of all indicators and reduces it into a fewer number of components

Each principal component PC_i is a new variable computed as a linear combination of the original (standardized) variables

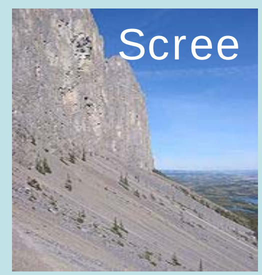
$$PC_1 = w_1 x_1 + w_2 x_2 + \cdots w_p x_p$$

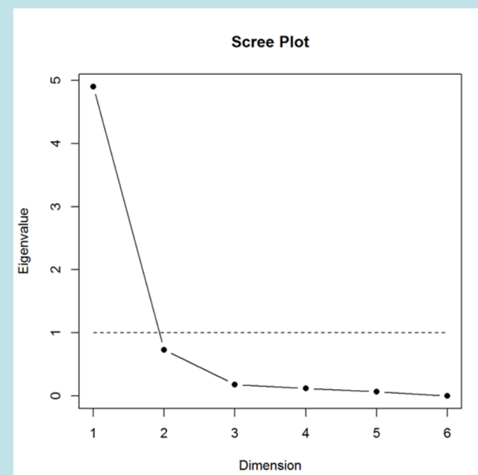
But...which is the magic number?

How many components do we keep in PCA?

Several methods exist. The 3 most common are:

1) Kaiser–Guttman 'Eigenvalues greater than one' criterion
(Guttman (1954), Kaiser (1960)). Select all components with eigenvalues over 1 (or 0.9)





2) Cattell's scree test

(Cattell (1966)) "Above the elbow" approach

3) Certain percentage of explained variance
e.g., >2/3, 75%, 80%,...

Weights based on Principal Component Analysis

- This method applies only when we have just one Principal Component (one-dim solution)
- The coefficients (standardised) of the first principal component are used as weights

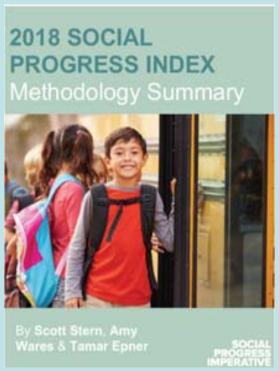
$$PC_1 = w_1 X_1 + w_2 X_2 + \ldots + w_n X_n$$

Weights

Empirical and objective option for weight selection

Good mathematical properties, determining the set of weights which explains the largest variation in the original indicators

PCA weights example: Social Progress Index



Basic Human Needs

Nutrition & Basic Medical Care
Undernourishment
Maternal mortality rate
Child mortality rate
Child stunting
Deaths from infectious diseases

Water & Sanitation
Access to at least basic drinking water
Access to piped water
Access to at least basic sanitation facilities
Rural open defecation

Shelter
Access to electricity
Quality of electricity supply
Household air pollution attributable deaths

Personal Safety
Homicide rate
Political killings and torture
Perceived criminality
Traffic deaths

Foundations of Wellbeing

Access to Basic Knowledge
Adult literacy rate
Primary school enrollment
Secondary school enrollment
Gender parity in secondary enrollment
Access to quality education

Access to Information & Communications
Mobile telephone subscriptions
Internet users
Participation in online governance
Access to independent media

Health & Wellness
Life expectancy at 60
Premature deaths from non-communicable diseases
Access to essential health services
Access to quality healthcare

Environmental Quality
Outdoor air pollution attributable deaths
Wastewater treatment
Greenhouse gas emissions2
Biome protection

Opportunity

Personal Rights
Political rights
Freedom of expression
Freedom of religion
Access to justice
Property rights for women

Personal Freedom & Choice
Vulnerable employment
Early marriage
Satisfied demand for contraception
Corruption

Inclusiveness
Acceptance of gays and lesbians
Discrimination and violence against minorities
Equality of political power by gender
Equality of political power by socioeconomic position
Equality of political power by social group

Access to Advanced Education
Years of tertiary schooling
Women's average years in school
Globally ranked universities
Percent of tertiary students enrolled in globally ranked universities

SOCIAL PROGRESS IMPERATIVE

Index
↕
3 Dimensions
↕
12 Components
↕
51 Indicators

PCA analysis for the indicators within each component

Source: Social Progress Index(2018)

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PCA weights example: Social Progress Index

Total variance explained

Component	Eigenvalue	Variance (%)	Cumulative variance (%)
PC1	2.28	57.11	57.11
PC2	0.87	21.81	78.92
PC3	0.51	12.80	91.71
PC4	0.33	8.29	100

Stopping criterion Eigenvalue > 1.0 (or cumulative variance over 0,70)

Environmental quality component

Pearson correlation coefficient between indicators and principal components

Indicators	PC1	PC2	PC3	PC4
Outdoor air pollution attributable deaths	0.88	-0.12	-0.11	-0.45
Wastewater treatment	0.83	-0.13	-0.44	0.32
Greenhouse gas emissions	0.78	-0.26	0.55	0.16
Biome protection	0.47	0.88	0.08	0.03

Weights scaled to unity

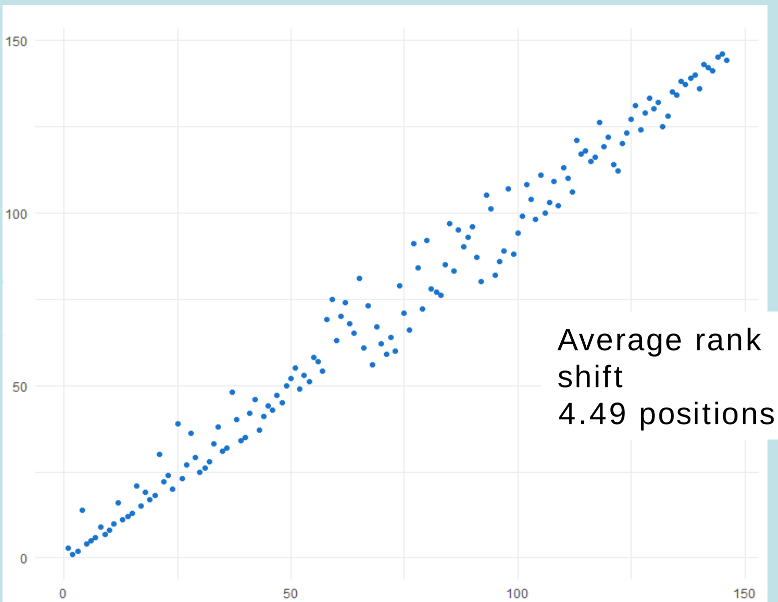
PC1*norm*	Equal Weights
0.30	0.25
0.28	0.25
0.26	0.25
0.16	0.25

$$PC_1 = 0.88\,X_1 + 0.83\,X_2 + 0.78\,X_3 + 0.47\,X_4$$

$$\boxed{PC_{1norm} = 0.30\,X_1 + 0.28\,X_2 + 0.26\,X_3 + 0.16\,X_4}$$

PCA weights example: Social Progress Index

Comparing weighting schemes
Environmental Quality Component

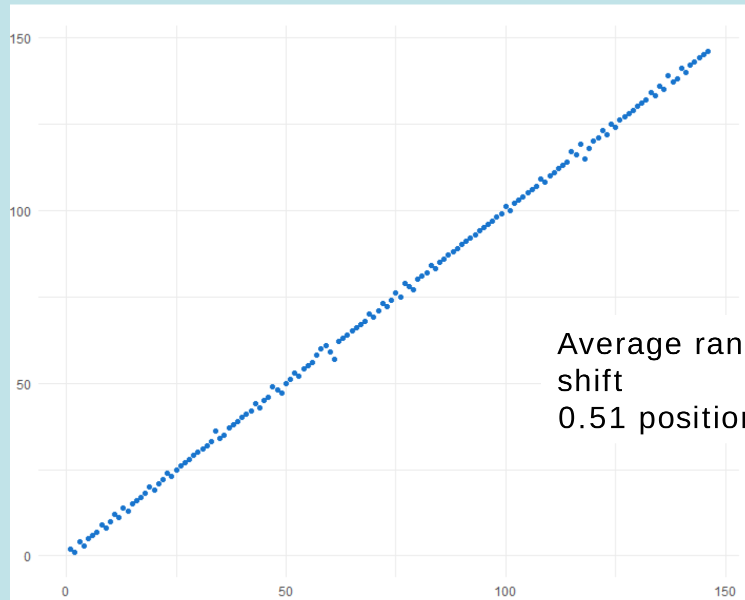


Equal weights

PCA weights

Average rank shift
4.49 positions

Comparing weighting schemes
SPI Index score



PCA weights

Average rank shift
0.51 positions

Similar rankings: choose the most simple!

Example: European Skills Index

Total variance explained

Component	Eigenvalue	Variance	Cumulative variance
PC1	$\boxed{1.75}$	58	58
PC2	0.88	29	88
PC3	0.37	12	100

Stopping criterion Eigenvalue > 1.0 (or cumulative variance over 0,70)

Pearson correlation coefficient

Pillars	PC1	PC2	PC3
Skills Development	0.88	-0.16	-0.44
Skills Activation	0.84	-0.36	0.41
Skills Matching	0.51	0.85	0.09

PC1*norm*
0.40
0.38
0.23

$$ESI = 0.40\ P1 + 0.38\ P2 + 0.23\ P3$$

Example: European Skills Index

Equal Weights

Pillars	Pearson Correlation Coefficient	R^2
Skills Development	0.80	0.64
Skills Activation	0.81	0.66
Skills Matching	0.64	0.41

PCA Weights

Pillars	Pearson Correlation Coefficient	R^2
Skills Development	0.84	0.70
Skills Activation	0.87	0.75
Skills Matching	0.52	0.27

The Index becomes even more unbalanced!

Con: small weights are assigned to indicators which are poorly correlated with others, irrespective of their possible related contextual importance ("Elitist index")

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Example: European Skills Index - Calibrated PCA weights

Pearson correlation coefficient

Pillars	**PC1**	**PC2**	**PC3**
Skills Development	0.88	-0.16	-0.44
Skills Activation	0.84	-0.36	0.41
Skills Matching	0.51	0.85	0.09

PCA*norm* Weights	Adjusted Weights
0.40	0.30
0.38	0.30
0.23	0.40

$$ESI = 0.30\ P_1 + 0.30\ P_2 + 0.40\ P_3$$

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Example: European Skills Index -
Calibrated PCA weights

Equal Weights

Pillars	Pearson Correlation Coefficient	R^2
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Adjusted Weights

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All Pillars are equally important!

Conclusions

Transparency is a key!
Weights have a strong impact on the final composite indicator score and on the resulting ranking. They should be explicit and transparent.

Opt for the simple!
Complicate approaches – more difficult to be communicated.

Test your selection!
The weighting method should always be tested using uncertainty and sensitivity analysis.

References

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