The European Commission's science and knowledge service

Joint Research Centre

European Commission

Step 5: Weighting methods (I)

Benefit of the Doubt (DEA approach)

Giulio Caperna

COIN 2019 - 17th JRC Annual Training on Composite Indicators & Scoreboards
04-06/11/2019, Ispra (IT)

European Commission

The International best player

To win the international championship…

…Please hit the Target

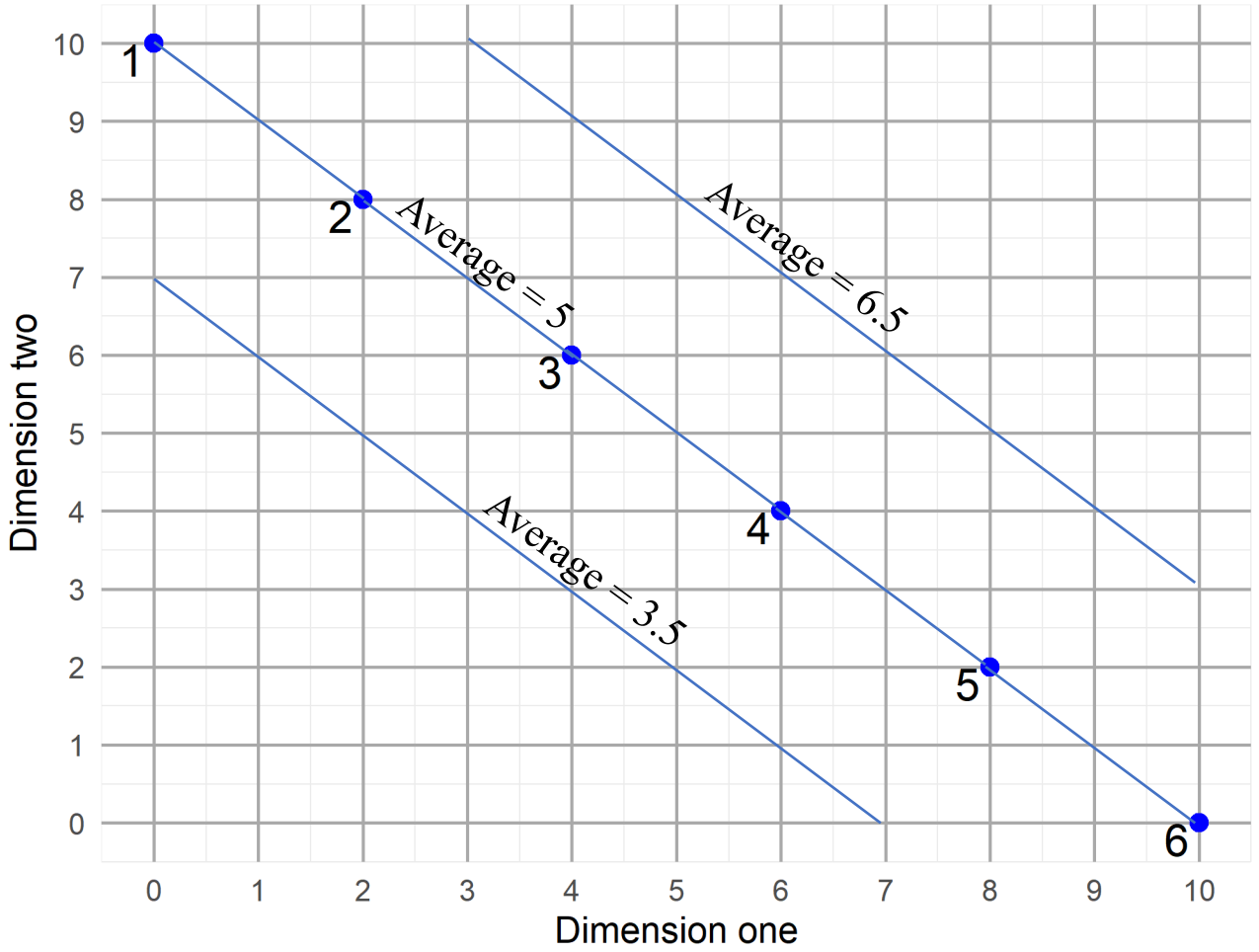
Somebody may object it is not fair

European Commission

The issue of weighting

- The selection of weights is a delicate task.
- Different points of view lead to different "correct" sets of weights.
- Comparing multi-dimensional performance on a fixed set of weights may not be fair to someone.

The issue of weighting, the frontier



Which one is the best?

- With equal weights ($\frac{1}{2}$ and $\frac{1}{2}$);

$$Id.1 = 0\times\frac{1}{2} + 10\times\frac{1}{2} = 5$$

$$Id.2 = 2\times\frac{1}{2} + 8\times\frac{1}{2} = 5$$

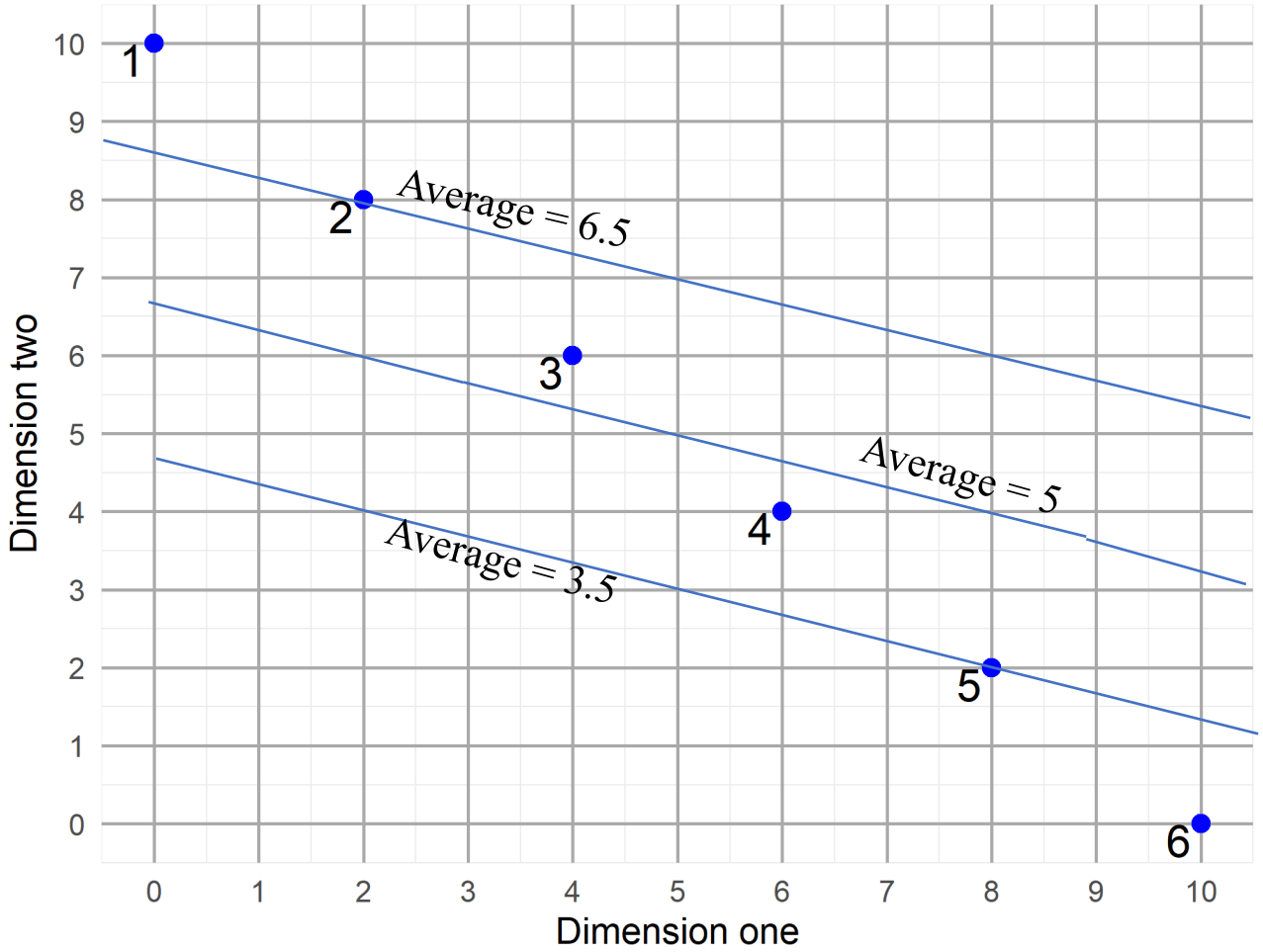
$$Id.3 = 4\times\frac{1}{2} + 6\times\frac{1}{2} = 5$$

$$Id.4 = 6\times\frac{1}{2} + 4\times\frac{1}{2} = 5$$

$$Id.5 = 8\times\frac{1}{2} + 2\times\frac{1}{2} = 5$$

$$Id.6 = 10\times\frac{1}{2} + 0\times\frac{1}{2} = 5$$

The issue of weighting, the frontier



Which one is the best?

- With unequal weights ($\frac{1}{4}$ and $\frac{3}{4}$);

$$Id.1 = 0\times\frac{1}{4} + 10\times\frac{3}{4} = 7.5$$

$$Id.2 = 2\times\frac{1}{4} + 8\times\frac{3}{4} = 6.5$$

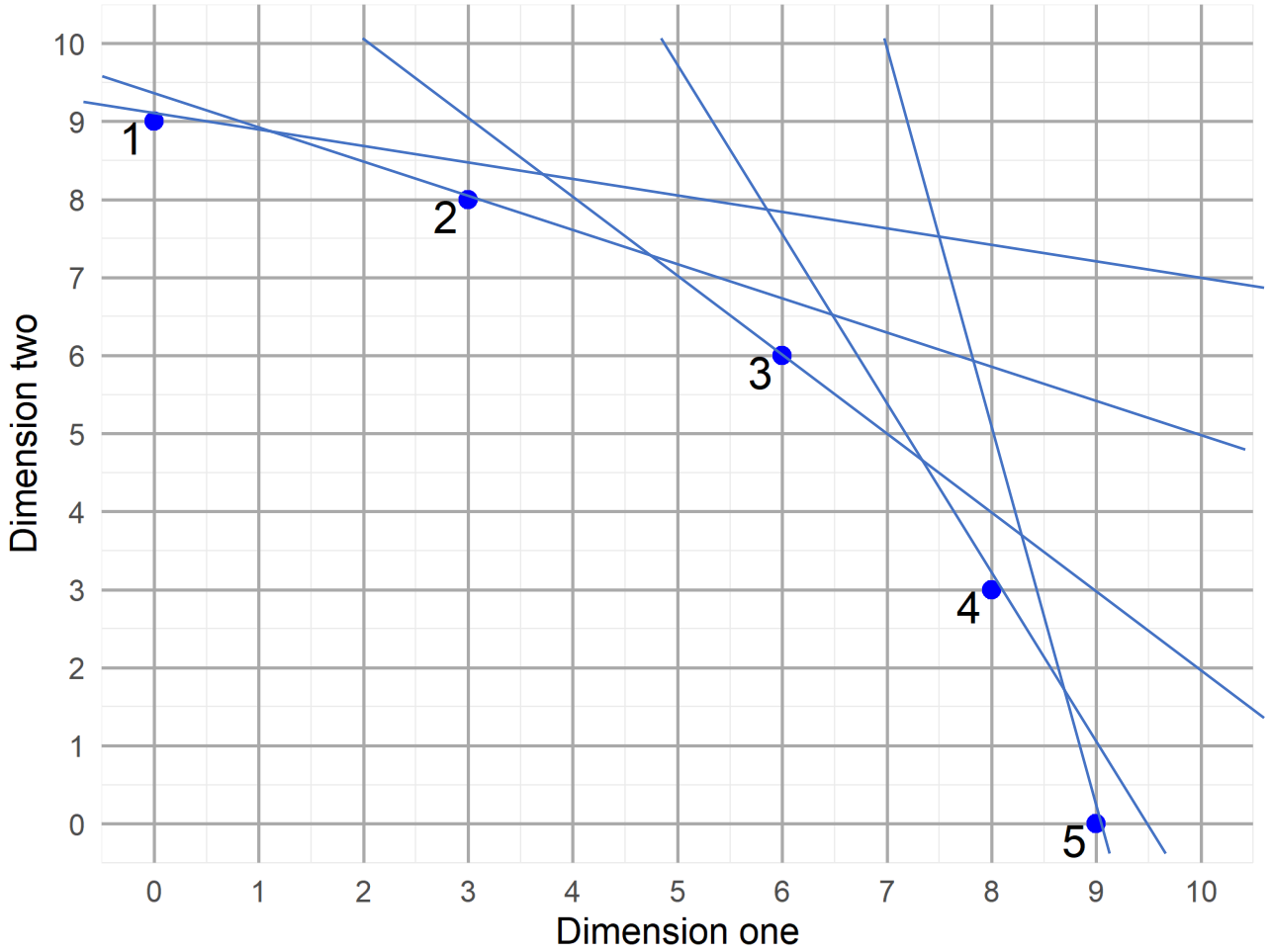
$$Id.3 = 4\times\frac{1}{4} + 6\times\frac{3}{4} = 5.5$$

$$Id.4 = 6\times\frac{1}{4} + 4\times\frac{3}{4} = 4.5$$

$$Id.5 = 8\times\frac{1}{4} + 2\times\frac{3}{4} = 3.5$$

$$Id.6 = 10\times\frac{1}{4} + 0\times\frac{3}{4} = 2.5$$

The issue of weighting, a personal frontier



Which one is the best?

- With case-specific weights;

$$\text{Id.1} = 0\times\frac{1}{5} + 9\times\frac{4}{5} = 7.2$$

$$\text{Id.2} = 3\times\frac{3}{8} + 8\times\frac{5}{8} = 6.125$$

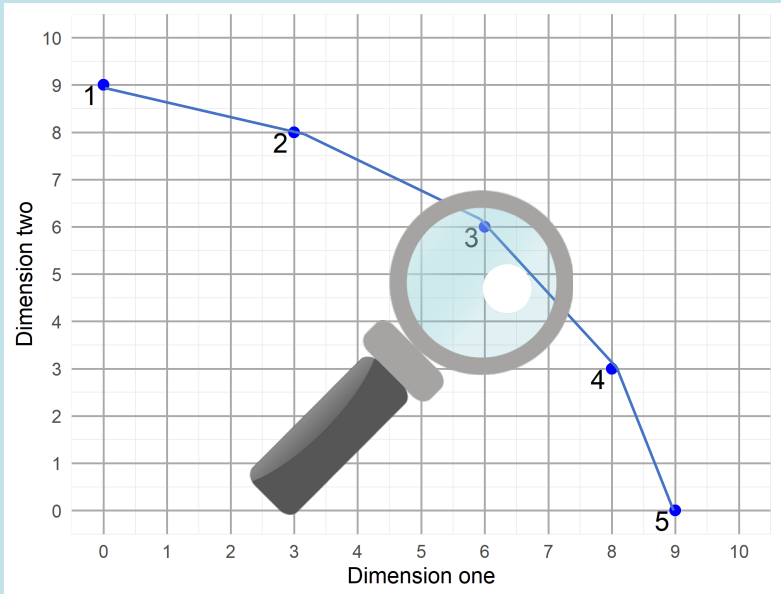
$$\text{Id.3} = 6\times\frac{1}{2} + 6\times\frac{1}{2} = 6$$

$$\text{Id.4} = 8\times\frac{5}{8} + 3\times\frac{3}{8} = 6.125$$

$$\text{Id.5} = 9\times\frac{4}{5} + 0\times\frac{1}{5} = 7.2$$

Classical Data Envelopment Analysis (DEA)

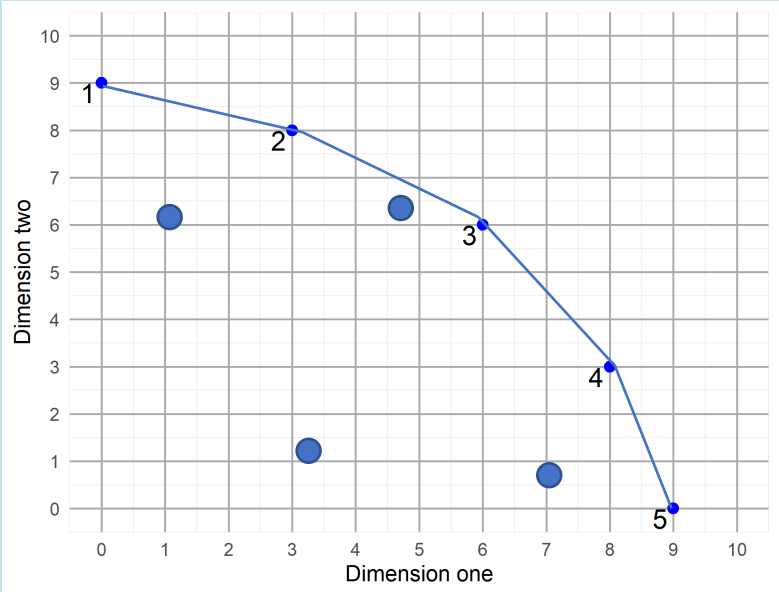
- Nonparametric method for measuring **efficiency**.
- Meant to measure **productivity** and compare firms and procedures: **outputs/inputs**.
- Weights are not known with certainty, many **sets of weights** are taken in consideration.



European Commission

Benefit of the Doubt (BoD)

- The use of DEA to **generate weights** of composite indicators
- Generating BoD weights **for every** observed unit
- Compare the performance of a unit with a **relative** benchmark



European Commission

Key steps (The math inside)

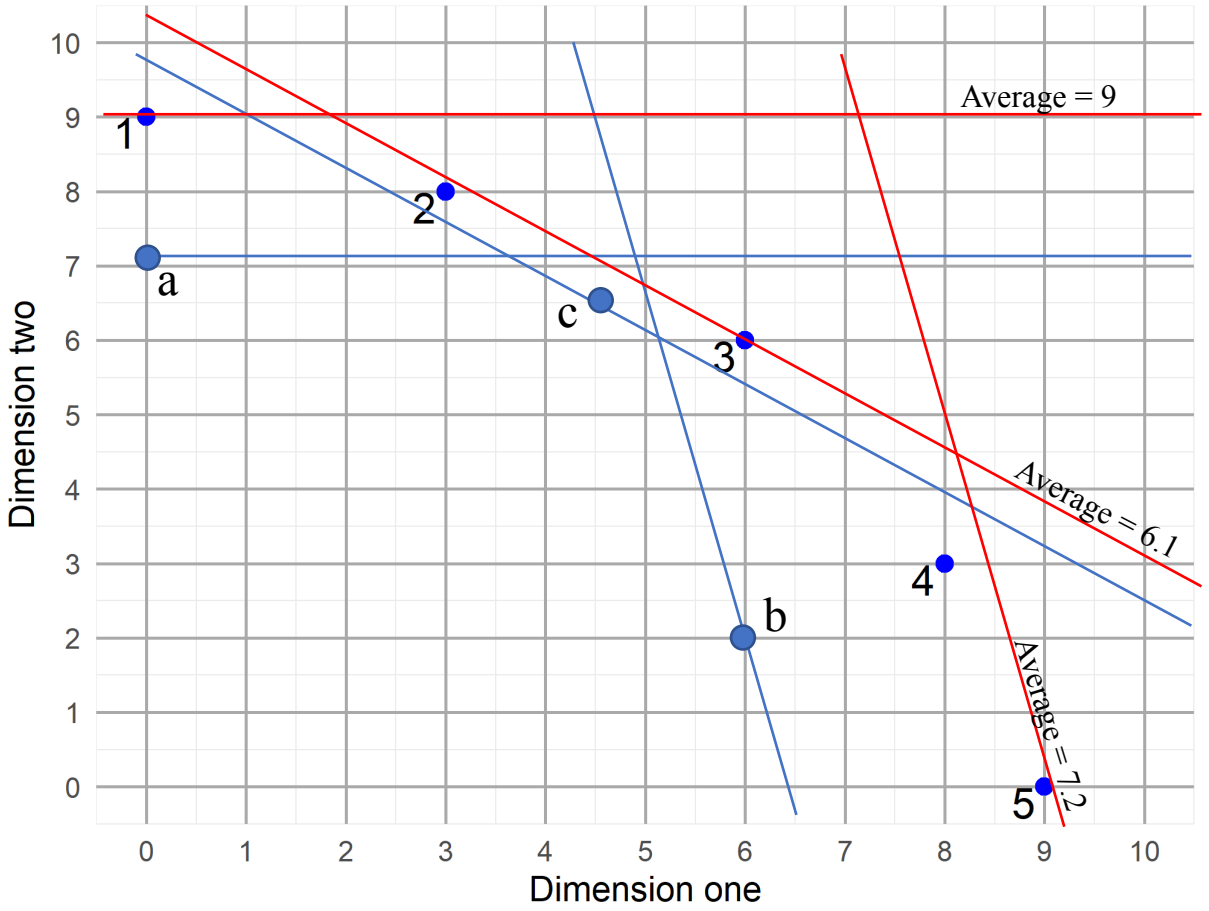
- Consider a country or unit $\hat{c}$.
- Identify highest-performing benchmark (=1).
- Find non-negative weights $w_{\hat{c}} = (w_{\hat{c}1}, w_{\hat{c}2}, \ldots, w_{\hat{c}I})$ that maximize unit $\hat{c}$'s performance relative to highest-performing benchmark.

$$B_{\hat{c}} = \max_{w_{\hat{c}} \geq 0} \frac{\sum_{i=1}^{I} w_{\hat{c}i} X_{\hat{c}i}}{\max_{c \in C} \sum_{i=1}^{I} w_{\hat{c}i} X_{ci}}$$

Weights that maximize $\hat{c}$'s result

Best-performing unit according to selected weights of unit $\hat{c}$

Benefit of the Doubt (BoD)



$B_a = 7.1/9.0 = 0.79$

$B_b = 5.2/7.2 = 0.72$

$B_c = 5.6/6.1 = 0.92$

BoD Scores represent the value respect to the relative benchmark

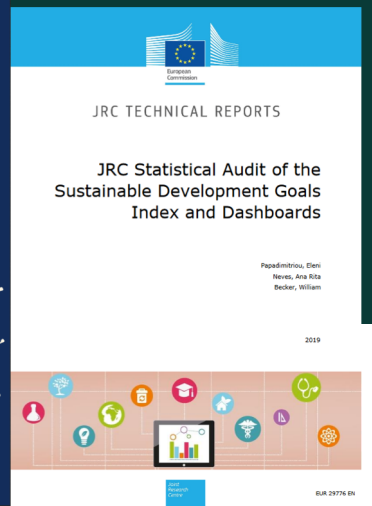
DEA weights and benchmarks: main features

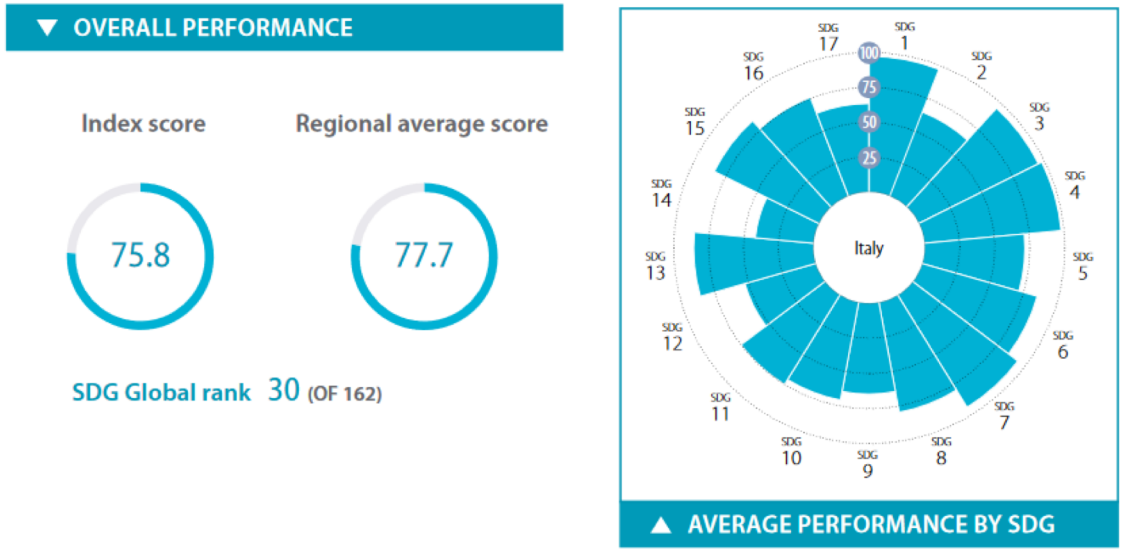
- DEA weights and benchmarks are endogenous and unit-specific.

- A unit can act as its own benchmark (i.e. not outperformed by another country given the weighting scheme).

- For instance, if a unit has the highest value on a certain indicator, then it automatically acts as its own benchmark and gets a score of 1.

Example: SDG Index and Dashboard 2019





All the Goals are aggregated with an Arithmetic mean with Equal Weights

European Commission

Example from the SDG Index and Dashboard

- Let's Look at the data
- How many countries are "the best" in one SDG?
- What is their BoD score?
- What about the others?
- Is one Goal sufficient to achieve a top score?

	Goal 2	Goal 8	Goal 12	BoD score
Canada	60.2	84.0	50.1	0.95
Germany	68.7	84.4	47.4	1.00
France	66.0	78.1	53.4	0.97
United Kingdom	66.4	82.9	42.9	0.97
Italy	64.3	78.7	51.7	0.94
Japan	68.0	88.5	55.6	1.00
Poland	61.2	84.4	73.7	1.00
Distopia	0.0	88.5	0.0	1.00

It may be too flexible

DEA weight restrictions

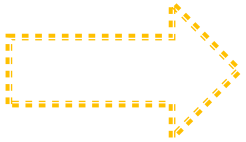
- It makes sense to restrict weights to lie within certain ranges.
- For each indicator i, we can impose weight's bounds:

$$L_i \leq w_i \leq U_i,$$

- The definition of the bounds is crucial in the BoD approach
- Then the sum of weights should always sum up to one

Scenario I: equal weights may vary ± 50%

	Nominal Weight	Lower Bound	Upper Bound
SDG 1	0.059	0.029	0.088
SDG 2	0.059	0.029	0.088
SDG 3	0.059	0.029	0.088
SDG 4	0.059	0.029	0.088
SDG 5	0.059	0.029	0.088
SDG 6	0.059	0.029	0.088
SDG 7	0.059	0.029	0.088
SDG 8	0.059	0.029	0.088
SDG 9	0.059	0.029	0.088
SDG 10	0.059	0.029	0.088
SDG 11	0.059	0.029	0.088
SDG 12	0.059	0.029	0.088
SDG 13	0.059	0.029	0.088
SDG 14	0.059	0.029	0.088
SDG 15	0.059	0.029	0.088
SDG 16	0.059	0.029	0.088
SDG 17	0.059	0.029	0.088



Country	SDG Index	SDG Index / Max Score
1 Denmark	85.2	85.2/85.2 = 1
2 Sweden	85.0	85.0/85.2 = 0.998
3 Finland	82.8	0.972
4 France	81.5	0.957
5 Austria	81.1	0.952
6 Germany	81.1	0.952
7 Czech Republic	80.7	0.947
8 Norway	80.7	0.947
9 Netherlands	80.4	0.944
10 Estonia	80.2	0.941
11 New Zealand	79.5	0.933
12 United Kingdom	79.4	0.932
13 Slovenia	79.4	0.932
14 Iceland	79.2	0.930
15 Belgium	78.9	0.926
16 Japan	78.9	0.926
17 Switzerland	78.8	0.925
… …	…	…
27 Italy	75.8	0.890
28 Chile	75.6	0.887
29 Lithuania	75.1	0.881
30 Luxembourg	74.8	0.878
31 United States	74.5	0.874
32 Australia	73.9	0.867
33 Israel	71.5	0.839
34 Greece	71.4	0.838
35 Mexico	68.5	0.804
36 Turkey	68.5	0.804

The DEA scores matrix : where would you expect to find “best” scores?

Every column is computed with different weights (according to reference country)

	Australia	Austria	Belgium	Canada	Switzerland	Chile	Czech Republic	Germany	Denmark	…	Lithuania	Luxembourg	Latvia	Mexico	…	Sweden	Turkey	United States
Australia	0.94	0.92	0.88	0.93	0.93	0.87	0.82	0.85	0.91	…	0.83	0.93	0.87	0.85	…	0.89	0.85	0.91
Austria	0.98	0.98	0.96	0.97	0.96	0.97	0.94	0.97	0.97	…	0.94	0.98	0.96	0.96	…	0.93	0.96	0.97
Belgium	0.92	0.93	0.96	0.94	0.9	0.91	0.94	0.95	0.93	…	0.92	0.92	0.93	0.92	…	0.89	0.94	0.93
Canada	0.95	0.95	0.93	0.97	0.93	0.93	0.91	0.92	0.93	…	0.92	0.94	0.92	0.94	…	0.9	0.93	0.95
Switzerland	0.97	0.96	0.94	0.96	0.98	0.94	0.9	0.96	0.96	…	0.9	0.97	0.91	0.94	…	0.92	0.93	0.96
Chile	0.94	0.94	0.89	0.94	0.9	0.99	0.94	0.93	0.87	…	0.94	0.92	0.95	0.97	…	0.87	0.96	0.92
Czech Republic	0.96	0.95	0.95	0.97	0.94	0.99	1	0.97	0.95	…	0.99	0.96	1	0.99	…	0.89	0.99	0.95
Germany	0.96	0.96	0.96	0.96	0.95	0.96	0.96	0.98	0.96	…	0.94	0.96	0.96	0.95	…	0.94	0.96	0.96
Denmark	1	1	1	1	1	1	1	1	1	…	0.99	1	1	1	…	0.98	1	1
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
Lithuania	0.91	0.91	0.9	0.93	0.87	0.95	0.95	0.91	0.87	…	0.96	0.91	0.96	0.95	…	0.82	0.94	0.9
Luxembourg	0.9	0.9	0.88	0.9	0.89	0.88	0.86	0.89	0.91	…	0.87	0.94	0.87	0.86	…	0.85	0.86	0.88
Latvia	0.92	0.92	0.92	0.94	0.89	0.96	0.97	0.94	0.91	…	0.97	0.92	0.97	0.96	…	0.84	0.96	0.91
Mexico	0.85	0.83	0.82	0.87	0.83	0.92	0.87	0.85	0.77	…	0.88	0.83	0.86	0.93	…	0.77	0.9	0.84
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
Sweden	1	0.99	1	1	1	0.99	0.98	0.99	1	…	0.97	0.99	0.99	1	…	1	1	0.99
Turkey	0.83	0.83	0.81	0.85	0.79	0.88	0.86	0.86	0.8	…	0.86	0.82	0.86	0.87	…	0.8	0.91	0.82
United States	0.93	0.92	0.91	0.93	0.92	0.9	0.89	0.9	0.87	…	0.89	0.91	0.91	0.89	…	0.87	0.88	0.94

	Australia	Austria	Belgium	Canada	Switzerland	Chile	Czech Republic	Germany	Denmark	…	Lithuania	Luxembourg	Latvia	Mexico	…	Sweden	Turkey	United States
Australia	**0.94**	0.92	0.88	0.93	0.93	0.87	0.82	0.85	0.91	…	0.83	0.93	0.87	0.85	…	0.89	0.85	0.91
Austria	0.98	**0.98**	0.96	0.97	0.96	0.97	0.94	0.97	0.97	…	0.94	0.98	0.96	0.96	…	0.93	0.96	0.97
Belgium	0.92	0.93	**0.96**	0.94	0.9	0.91	0.94	0.95	0.93	…	0.92	0.92	0.93	0.92	…	0.89	0.94	0.93
Canada	0.95	0.95	0.93	**0.97**	0.93	0.93	0.91	0.92	0.93	…	0.92	0.94	0.92	0.94	…	0.9	0.93	0.95
Switzerland	0.97	0.96	0.94	0.96	**0.98**	0.94	0.9	0.96	0.96	…	0.9	0.97	0.91	0.94	…	0.92	0.93	0.96
Chile	0.94	0.94	0.89	0.94	0.9	**0.99**	0.94	0.93	0.87	…	0.94	0.92	0.95	0.97	…	0.87	0.96	0.92
Czech Republic	0.96	0.95	0.95	0.97	0.94	0.99	**1**	0.97	0.95	…	0.99	0.96	1	0.99	…	0.89	0.99	0.95
Germany	0.96	0.96	0.96	0.96	0.95	0.96	0.96	**0.98**	0.96	…	0.94	0.96	0.96	0.95	…	0.94	0.96	0.96
Denmark	1	1	1	1	1	1	1	1	**1**	…	0.99	1	1	1	…	0.98	1	1
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
Lithuania	0.91	0.91	0.9	0.93	0.87	0.95	0.95	0.91	0.87	…	**0.96**	0.91	0.96	0.95	…	0.82	0.94	0.9
Luxembourg	0.9	0.9	0.88	0.9	0.89	0.88	0.86	0.89	0.91	…	0.87	**0.94**	0.87	0.86	…	0.85	0.86	0.88
Latvia	0.92	0.92	0.92	0.94	0.89	0.96	0.97	0.94	0.91	…	0.97	0.92	**0.97**	0.96	…	0.84	0.96	0.91
Mexico	0.85	0.83	0.82	0.87	0.83	0.92	0.87	0.85	0.77	…	0.88	0.83	0.86	**0.93**	…	0.77	0.9	0.84
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
Sweden	1	0.99	1	1	1	0.99	0.98	0.99	1	…	0.97	0.99	0.99	1	…	**1**	1	0.99
Turkey	0.83	0.83	0.81	0.85	0.79	0.88	0.86	0.86	0.8	…	0.86	0.82	0.86	0.87	…	0.8	**0.91**	0.82
United States	0.93	0.92	0.91	0.93	0.92	0.9	0.89	0.9	0.87	…	0.89	0.91	0.91	0.89	…	0.87	0.88	**0.94**

Scenario II: allow the weights to reach zero (± 100%)

	Nominal Weight	Lower Bound	Upper Bound
SDG 1	0.059	0	0.118
SDG 2	0.059	0	0.118
SDG 3	0.059	0	0.118
SDG 4	0.059	0	0.118
SDG 5	0.059	0	0.118
SDG 6	0.059	0	0.118
SDG 7	0.059	0	0.118
SDG 8	0.059	0	0.118
SDG 9	0.059	0	0.118
SDG 10	0.059	0	0.118
SDG 11	0.059	0	0.118
SDG 12	0.059	0	0.118
SDG 13	0.059	0	0.118
SDG 14	0.059	0	0.118
SDG 15	0.059	0	0.118
SDG 16	0.059	0	0.118
SDG 17	0.059	0	0.118

Country	SDG Index	SDG Index / Max Score
1 Denmark	85.2	1.000
2 Sweden	85	0.998
3 Finland	82.8	0.972
4 France	81.5	0.957
5 Austria	81.1	0.952
6 Germany	81.1	0.952
7 Czech Republic	80.7	0.947
8 Norway	80.7	0.947
9 Netherlands	80.4	0.944
10 Estonia	80.2	0.941
11 New Zealand	79.5	0.933
12 United Kingdom	79.4	0.932
13 Slovenia	79.4	0.932
14 Iceland	79.2	0.930
15 Belgium	78.9	0.926
16 Japan	78.9	0.926
17 Switzerland	78.8	0.925
……		
27 Italy		
28 Chile		
29 Lithuania	75.1	0.881
30 Luxembourg	74.8	0.878
31 United States	74.5	0.874
32 Australia	73.9	0.867
33 Israel	71.5	0.839
34 Greece	71.4	0.838
35 Mexico	68.5	0.804
36 Turkey	68.5	0.804

Ok, good, now what?

Do we expect less or more “one”?

European Commission

Cross-efficiency DEA

- Calculate the score of each unit $|C|$ times (where $|C|$ is number of units), using the optimal weights of each unit.

- The average efficiency score is usually then used to rank countries (but a median or something else may be good as well)

- While classical DEA cannot provide a clear ranking, cross-efficiency DEA can (Sexton et al., 1986).

Cross Efficiency in Scenario I

	Australia	Austria	Belgium	Canada	…	Lithuania	Luxembourg	Latvia	Mexico	…	Sweden	Turkey	United States	Cross Efficiency (mean)	Rank
Australia	**0.94**	0.92	0.88	0.93	…	0.83	0.93	0.87	0.85	…	0.89	0.85	0.91	0.881	31
Austria	0.98	**0.98**	0.96	0.97	…	0.94	0.98	0.96	0.96	…	0.93	0.96	0.97	0.962	7
Belgium	0.92	0.93	**0.96**	0.94	…	0.92	0.92	0.93	0.92	…	0.89	0.94	0.93	0.929	24
Canada	0.95	0.95	0.93	**0.97**	…	0.92	0.94	0.92	0.94	…	0.9	0.93	0.95	0.934	22
Switzerland	0.97	0.96	0.94	0.96	…	0.9	0.97	0.91	0.94	…	0.92	0.93	0.96	0.939	19
Chile	0.94	0.94	0.89	0.94	…	0.94	0.92	0.95	0.97	…	0.87	0.96	0.92	0.931	23
Czech Republic	0.96	0.95	0.95	0.97	…	0.99	0.96	1	0.99	…	0.89	0.99	0.95	0.970	6
Germany	0.96	0.96	0.96	0.96	…	0.94	0.96	0.96	0.95	…	0.94	0.96	0.96	0.958	8
Denmark	1	1	1	1	…	0.99	1	1	1	…	0.98	1	1	0.998	1
…	…	…	…	…	…	…	…	…	…	…	…	…	…		
Lithuania	0.91	0.91	0.9	0.93	…	**0.96**	0.91	0.96	0.95	…	0.82	0.94	0.9	0.917	28
Luxembourg	0.9	0.9	0.88	0.9	…	0.87	**0.94**	0.87	0.86	…	0.85	0.86	0.88	0.878	32
Latvia	0.92	0.92	0.92	0.94	…	0.97	0.92	**0.97**	0.96	…	0.84	0.96	0.91	0.935	21
Mexico	0.85	0.83	0.82	0.87	…	0.88	0.83	0.86	**0.93**	…	0.77	0.9	0.84	0.859	35
…	…	…	…	…	…	…	…	…	…	…	…	…	…		
Sweden	1	0.99	1	1	…	0.97	0.99	0.99	1	…	**1**	1	0.99	0.994	2
Turkey	0.83	0.83	0.81	0.85	…	0.86	0.82	0.86	0.87	…	0.8	**0.91**	0.82	0.842	36
United States	0.93	0.92	0.91	0.93	…	0.89	0.91	0.91	0.89	…	0.87	0.88	**0.94**	0.900	30

The results take every point of view into account

Easy to defend

Good with no consensus on weights

“I’ll give you the benefit of the doubt, and follow your recipe for Lasagna”

Cit. Somebody
(Not my Grandmother)

Any questions?

You can find us at jrc-coin@ec.europa.eu

European Commission

References

- Amado, C.A.F., Sao José, J. M. S., & Santos, S. P. (2016). Measuring active ageing: A Data Envelopment Analysis approach. *European Journal of Operational Research*, 255, 207-223.
- Boyd, S., & Vandenberghe, L. (2004). *Convex Optimization*. Cambridge University Press.
- Cherchye, L., Knox Lovell, C. A., Moesen, W., & Van Puyenbroeck, T. (2007). One market, one number? A composite indicator assessment of EU internal market dynamics. *European Economic Review*, 51(3), 749-779.
- Cherchye, L., Moesen, W., Rogge, N., & Van Puyenbroeck, T. (2007). An introduction to 'benefit of the doubt' composite indicators. *Social Indicators Research*, 82(1), 111-145.
- Cherchye, L., Moesen, W., Rogge, N., Van Puyenbroeck, T., Saisana, M., Saltelli, A., Liska, R. & Tarantola, S. (2008). Creating composite indicators with DEA and robustness analysis: the case of the technology achievement index. *Journal of the Operational Research Society*, 59(2), 239-251.
- Cooper, W. W., Park, K. S., & Yu, G. (1999). IDEA and AR-IDEA: Models for dealing with imprecise data in DEA. *Management Science*, 45(4), 597-607.
- Despotis, D. K. (2005). A reassessment of the human development index via data envelopment analysis. *Journal of the Operational Research Society*, 56(8), 969-980.
- Despotis, D. K., & Smirlis, Y. G. (2002). Data envelopment analysis with imprecise data. *European Journal of Operational Research*, 140(1), 24-36.
- Sachs, J., Schmidt-Traub, G., Kroll, C., Lafortune, G., Fuller, G. (2019): Sustainable Development Report 2019. New York: Bertelsmann Stiftung and Sustainable Development Solutions Network (SDSN)
- Vidoli, F., Fusco, E.(2019): Compind: Composite Indicators Functions. R package version 2.1.
- Zanella, A., Camanho, A. S., & Dias, T. G. (2015). Undesirable outputs and weighting schemes in composite indicators based on data envelopment analysis. *European Journal of Operational Research*, 245, 517-530.
- Zhou, P., Ang, B. W., & Poh, K. L. (2007). A mathematical programming approach to constructing composite indicators. *Ecological Economics*, 62(2), 291-297.

Appendix

DEA and linear programming

- **Essential:** underlying optimization problem has a linear-fractional structure and thus can be transformed into an equivalent linear program (Boyd and Vandenberghe, 2004).

$$\max_{w_{\hat{c}} \geq 0} \sum_{i=1}^{I} w_{\hat{c}i} X_{\hat{c}i}$$

$$\text{s.t.} \quad \sum_{i=1}^{I} w_{\hat{c}i} X_{ci} \leq 1, \quad \forall c \in C$$

- This allows for tractable and efficient computation.

Advantages of DEA

- Endogenous weighting adds flexibility: weights adapt to measurement units.

- Sub-indicator shares are pure numbers, i.e. independent of the units of measurement of the sub-indicators.

- Other kinds of *a priori* weight restrictions can also be considered.