The European Commission’s science and knowledge service

Joint Research Centre
Step 6: Weighting methods (II)

Budget allocation, Analytic Hierarchy Process

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06-08/11/2017, Ispra (IT)
Decalogue

Step 10. Presentation & dissemination
Step 9. Association with other variables
Step 8. Back to the indicators
Step 7. Robustness & sensitivity
Step 6. Weighting & aggregation
Step 5. Data normalisation
Step 4. Multivariate analysis
Step 3. Data treatment
Step 2. Selection of indicators
Step 1. Conceptual framework
Weights

- Equal weights

Weights based on statistical models
- Principal component/Factor analysis
- Data envelopment analysis
- Regression approach
- Unobserved components models

Weights based on participation
- Budget allocation
- Analytic hierarchy process
- Conjoint analysis
Phases

1. Selection of experts/stakeholders for the evaluation;
   a. Number
   b. Background/Expertise
2. Allocation of budget to indicators;
3. Calculation of weights;
4. Iteration of the budget allocation until convergence is reached (optional)

Suited for up to 8-10 indicators
Source: *Quantifying the qualitative: Eliciting expert input to develop the Multidimensional Poverty Assessment Tool* (Cohen, Saisana, J of Dev. Studies, 2014, 50(1))

**Example 1: Multidimensional Poverty Assessment Tool, Weights based on 42 experts**

- **Purpose:** Eliciting weights to be assigned to the indicators within each component

- **Selection of Experts**
  - 42 Experts from 10 countries and 28 organizations
  - Mainly from UN agencies and universities
  - Selection based on expertise on poverty assessment tools in developing countries

- **No real sampling frame**
MPAT - Results of the Budget Allocation

Example 1: Multidimensional Poverty Assessment Tool, Weights based on 42 experts

In 4 Components, the average expert weights are similar to equal weighting.
Example 1: Multidimensional Poverty Assessment Tool, Weights based on 42 experts

MPAT - Results of the Budget Allocation

India (n=21)
Rest of the World (16)
Example 1: Multidimensional Poverty Assessment Tool, Weights based on 42 experts

In 10 subcomponents the average expert opinion from China is very different than that of India or the Rest of the world.
Example 2: The Cultural and Creative Cities Monitor, 2017 Edition

- European Capitals of Culture: 93
- UNESCO Creative Cities: 22
- Cities hosting at least two international cultural festivals: 53
### Purpose:
Eliciting weights to be assigned to
- the 3 sub-indices
- the 9 dimensions

### Selection of Experts
- **17 Experts**
  - 5 from EC, 6 from Academia, 6 from international organisations

### When
- Second participatory workshop of the C3 Monitor - November 2016

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### Table: C3 Index - Snapshot

<table>
<thead>
<tr>
<th>Weight</th>
<th>Sub-indices</th>
<th>Dimensions</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>3.3 Local &amp; International Connections</td>
<td>3.3.1 Distance from cities</td>
<td>26. Underemployment, 27. Potential road accessibility, 28. Direct trains to other cities, 29. Quality of governance</td>
</tr>
</tbody>
</table>

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**Example 2: The Cultural and Creative Cities Monitor, 2017 Edition**
Example 2: The Cultural and Creative Cities Monitor, 2017 Edition

Weights assigned to the three sub-indices by each group

«Enabling Environment» sub-index
- Emerged from the discussion that accessibility and governance dimensions should have a minimum weight

<table>
<thead>
<tr>
<th>Sub-index</th>
<th>Group 3</th>
<th>Group 2</th>
<th>Group 1</th>
<th>Average of the 3 groups</th>
<th>Final Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cultural Vibrancy</td>
<td>40</td>
<td>50</td>
<td>40</td>
<td>43.3</td>
<td>40</td>
</tr>
<tr>
<td>Creative Economy</td>
<td>40</td>
<td>30</td>
<td>35</td>
<td>35.0</td>
<td>40</td>
</tr>
<tr>
<td>Enabling Environment</td>
<td>20</td>
<td>20</td>
<td>25</td>
<td>21.7</td>
<td>20</td>
</tr>
</tbody>
</table>

Human Capital & Education - Academic Appeal: 40
Openness, Tolerance and Trust: 40
Accessibility - local & international: 15
Governance & Regulations: 5
Suggestions for the BAL

- When possible- use a **sampling frame** to select the experts & **maximize response rate**
  → *Compensating* experts might increase participation (Chowdury and Squire, 2006)

- **Experts** with balance of **diverse backgrounds**

- Collect information on the **characteristics** of the experts (Cooke, 1991)

- During the survey, **do not bother about the “100 points” sum** when there are more than 4 indicators (rescale to 100 after the survey).

- **Randomize** the order of the components, so that some experts evaluate first component A and others component B, and so on.
Developed by Thomas Saaty (1980, 1987)

Multi-criteria decision making method

Phases
1. Selection of experts/stakeholders for the evaluation;
2. Pairwise comparisons of indicators on a scale 1 to 9 (1: equally important, 9: most important);
3. Calculation of weights through the derivation of the priority vector;
4. Estimation of consistency of the experts’ assessment.

Recommended for less than 10 indicators
Phase 2- **PAIRWISE COMPARISONS** to express THE RELATIVE IMPORTANCE OF ONE INDICATOR OVER ANOTHER.

Which indicator do you feel is more important?

1 EQUAL  3 MODERATE  5 STRONG  7 VERY STRONG  9 EXTREME

Indicator 1

Indicator 2

Preference for I1

Preference for I2

Indicator 1

Indicator 2

Indicator 3

Indicator 3
Phase 2- **PAIRWISE COMPARISONS** to express THE RELATIVE IMPORTANCE OF ONE INDICATOR OVER ANOTHER

1 EQUAL  3 MODERATE  5 STRONG  7 VERY STRONG  9 EXTREME

Set up a $n \times n$ matrix (A) with $n$ being the number of indicators

<table>
<thead>
<tr>
<th>Matrix A</th>
<th>Indicator 1</th>
<th>Indicator 2</th>
<th>Indicator 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indicator1</td>
<td>1</td>
<td>1/3</td>
<td>5</td>
</tr>
<tr>
<td>Indicator 2</td>
<td>3</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>Indicator 3</td>
<td>1/5</td>
<td>1/7</td>
<td>1</td>
</tr>
</tbody>
</table>

More time consuming than budget allocation: $n \cdot (n-1)/2$ comparisons needed

I$_2$ three times more important than I$_1$

I$_{21}$=3

I$_{12}$ Reciprocal value of I$_{21}$
Phase 3- Calculation of WEIGHTS

For each matrix A, need to derive the weights

→ different methods

Saaty (1990) shows that the weight vector is the eigenvector of the matrix A corresponding to the highest eigenvalue.

\( \lambda \) is called an eigenvalue if there is a nonzero vector \( x \) such that \( Ax = \lambda x \). \( x \) is called an eigenvector of \( A \) corresponding to \( \lambda \).
### Analytic Hierarchy Process

**Phase 3-** Proxy of the **weights vector** when the number of Indicators is limited –normalized columns method

#### a - Sum each column of the matrix

<table>
<thead>
<tr>
<th></th>
<th>I1</th>
<th>I2</th>
<th>I3</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>1</td>
<td>1/3</td>
<td>5</td>
</tr>
<tr>
<td>I2</td>
<td>3</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>I3</td>
<td>1/5</td>
<td>1/7</td>
<td>1</td>
</tr>
<tr>
<td>Sum</td>
<td>21/5</td>
<td>31/21</td>
<td>13</td>
</tr>
</tbody>
</table>

#### b - Normalized relative weights

<table>
<thead>
<tr>
<th></th>
<th>I1</th>
<th>I2</th>
<th>I3</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>5/21</td>
<td>7/31</td>
<td>5/13</td>
</tr>
<tr>
<td>I2</td>
<td>15/21</td>
<td>21/31</td>
<td>7/13</td>
</tr>
<tr>
<td>I3</td>
<td>1/21</td>
<td>3/31</td>
<td>1/13</td>
</tr>
<tr>
<td>Sum</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

#### c - Average across the rows

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>0.2828</td>
</tr>
<tr>
<td>I2</td>
<td>0.6434</td>
</tr>
<tr>
<td>I3</td>
<td>0.0738</td>
</tr>
</tbody>
</table>

Weights
Phase 3- Other method to retrieve the weights vector – geometric mean method

<table>
<thead>
<tr>
<th></th>
<th>I1</th>
<th>I2</th>
<th>I3</th>
<th>geometric mean</th>
<th>normalized geometric mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>1.00</td>
<td>0.33</td>
<td>5.00</td>
<td>1.19</td>
<td>0.28</td>
</tr>
<tr>
<td>I2</td>
<td>3.00</td>
<td>1.00</td>
<td>7.00</td>
<td>2.76</td>
<td>0.65</td>
</tr>
<tr>
<td>I3</td>
<td>0.20</td>
<td>0.14</td>
<td>1.00</td>
<td>0.31</td>
<td>0.07</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
<td>4.25</td>
<td></td>
</tr>
</tbody>
</table>
Analytic Hierarchy Process

Phase 4 - Estimation of consistency ratio

- Experts’ assessment: are they consistent?
  Consistency: $I_{13} = I_{12} \times I_{23}$

- Experts’ assessment = subjective preferences
  - Some inconsistencies are acceptable

- For each expert: necessary to compute a consistency ratio

  \[ CR = \frac{\text{consistency index of matrix } A}{\text{consistency of a random-like matrix}} = \frac{\text{CI}(A)}{\text{CR}_n} \]

  - Suggested rule-of-thumb is $CR \leq 0.1$ although $0.2$ is often cited – do not drastically affect the weights (Saaty, 1980).

<table>
<thead>
<tr>
<th></th>
<th>I1</th>
<th>I2</th>
<th>I3</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>1</td>
<td>1/3</td>
<td>5</td>
</tr>
<tr>
<td>I2</td>
<td>3</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>I3</td>
<td>1/5</td>
<td>1/7</td>
<td>1</td>
</tr>
</tbody>
</table>

- Measure how a country is creating and diffusing new & existent technologies and building a human skill base with 8 achievement indicators

- Original CI: equal weight

- Departure from the original weighting scheme using an AHP based on a survey of 20 scientists of the JRC
## Example 1: Technological Achievement Index

### Using Pairwise Comparisons to Express the Relative Importance of One Criterion Over Another

**Patents** | **Royalties** | **Internet** | **Tech. Exports** | **Telephones** | **Electricity** | **Schooling** | **University St.**
---|---|---|---|---|---|---|---
Patents | 1 | 1/3 | 5 | 4 | 3 | 9 | 1/6 | 1/8
Royalties | 3 | 1 | 3 | 1/4 | 5 | 9 | 1/3 | 1/4
Internet | 1/5 | 1/3 | 1 | 1/6 | 2 | 2 | 1/7 | 1/6
Tech. Exports | 1/4 | 4 | 6 | 1 | 5 | 9 | 1/4 | 1/5
Telephones | 1/3 | 1/5 | 1/2 | 1/5 | 1 | 7 | 1/9 | 1/9
Electricity | 1/9 | 1/9 | 1/2 | 1/9 | 1/7 | 1 | 1/9 | 1/9
Schooling | 6 | 3 | 7 | 4 | 9 | 9 | 1 | 2
University St. | 8 | 4 | 6 | 5 | 9 | 9 | 1/2 | 1

**Note:**
- The reciprocal matrix $A$ for 1 expert.
- Example 1: Technological Achievement Index.

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22 JRC-COIN © | Step 5: Weighting methods (II) Budget allocation, Analytic Hierarchy Process
TAI – Reciprocal matrix A - any inconsistency?

Example 1: Technological Achievement Index

<table>
<thead>
<tr>
<th>Patents</th>
<th>Royalties</th>
<th>Internet</th>
<th>Tech.Exports</th>
<th>Telephones</th>
<th>Electricity</th>
<th>Schooling</th>
<th>University St.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patents</td>
<td>1</td>
<td>1/3</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>9</td>
<td>1/6</td>
</tr>
<tr>
<td>Royalties</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1/4</td>
<td>1/3</td>
<td>1/3</td>
<td>1/4</td>
</tr>
<tr>
<td>Internet</td>
<td>1/5</td>
<td>1/3</td>
<td>1</td>
<td>1/6</td>
<td>1/4</td>
<td>1/7</td>
<td>1/6</td>
</tr>
<tr>
<td>Tech.Exports</td>
<td>1/4</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>1/4</td>
<td>1/4</td>
<td>1/5</td>
</tr>
<tr>
<td>Telephones</td>
<td>1/3</td>
<td>1/5</td>
<td>1/2</td>
<td>1/5</td>
<td>1</td>
<td>7</td>
<td>1/9</td>
</tr>
<tr>
<td>Electricity</td>
<td>1/9</td>
<td>1/9</td>
<td>1/2</td>
<td>1/9</td>
<td>1/7</td>
<td>1</td>
<td>1/9</td>
</tr>
<tr>
<td>Schooling</td>
<td>6</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>9</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>University St.</td>
<td>8</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>9</td>
<td>9</td>
<td>1/2</td>
</tr>
</tbody>
</table>

For a matrix of size $Q \times Q$, only $Q-1$ comparisons are required to establish weights for $Q$ indicators. But the number of AHP comparisons is $Q(Q-1)/2$. 

Try to spot the inconsistency…
Example 1: Technological Achievement Index

Which Indicator Do You Feel Is More Important?

To What Degree?

Patents 1 1/3 5 4 3 9 1/6 1/8
Royalties 3 1 3 1/4 5 9 1/3 1/4
Internet 1/5 1/3 1 1/6 2 2 1/7 1/6
Tech. Exports 1/4 4 6 1 5 9 1/4 1/5
Telephones 1/3 1/5 1/2 1/5 1 7 1/9 1/9
Electricity 1/9 1/9 1/2 1/9 1/7 1 1/9 1/9
Schooling 6 3 7 4 9 9 1 2
Univ. St. 8 4 6 5 9 9 1/2 1

Weights
- Patents 0.109
- Royalties 0.103
- Internet hosts 0.029
- Tech. Exports 0.117
- Telephones 0.030
- Electricity 0.014
- Schooling 0.301
- Univ. St. 0.297

Inconsistency 17.4 %
Example 1: Technological Achievement Index

Inconsistencies range from 1.1% - 45.5%
(desired < 10-20%)
Weights obtained by **Budget allocation** are closer to equal weights than those obtained by **Analytic Hierarchy Process**.
AHP – other examples

Example 2: Gender Equality Index

AHP

Purpose: Assign weights at the domain level

Experts: EIGE’s Working Group on the Gender Equality Index and EIGE’s Expert Forum. Experts’ Final weight at the domain level: expert weights average

Response rate 50%
Based on consistency ratio, 60% of experts weights kept
Conjoint analysis - CA

- BAL and AHP possible when limited numbers of dimensions/indicators
- Alternatively, expert-based weights can be derived from **conjoint analysis** (CA)
  - Respondents rank “alternative scenarios” (Hair *et al.*, 1995)
    - Each scenario → different values of the indicators/dimensions
  - Approach frequently used in **marketing** and **consumer** research
  - **Decompositional** multivariate data analysis.
References


References

Any questions?
Welcome to email us at: jrc-coin@ec.europa.eu

COIN in the EU Science Hub

COIN tools are available at:
https://composite-indicators.jrc.ec.europa.eu/