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Step 6: Aggregation rules

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Outline of Step 6

- Additive and multiplicative aggregations
- Methods based on ranks
- Methods based on pairwise comparisons

Additive and multiplicative aggregations

Normalise data and treat outliers for each (*quantitative*) indicator as a starting point

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Additive aggregations

The ***arithmetic mean*** of a list of *n* real numbers equals:

$$\frac{1}{n}\sum_{i=1}^{n} x_i$$

This is the simplest, most obvious and most widespread aggregation method

<u>Key underlying property</u>: Perfect substitutability – compensates bad performance in one with good in another

Arithmetic mean

The score corresponding to the 4th pillar
of the GTCI index in country *i* is calculated as
The arithmetic avergate of sub-pillars 4.1 and 4.2:

Sustainability score = 37.04
Lifestyle score = 59.60

Retain pillar score = ½ (37.04+59.60) = 48.32

4	**RETAIN**	**48.32**	**70**
4.1	Sustainability	37.04	78
4.1.1	Pension system	37.37	56
4.1.2	Taxation	43.58	73
4.1.3	Brain retention	30.16	94
4.2	Lifestyle	59.60	64
4.2.1	Environmental performance	69.58	57
4.2.2	Personal safety	62.00	56
4.2.3	Physician density	14.55	78
4.2.4	Sanitation	92.27	57

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Multiplicative aggregations

The ***geometric mean*** of a list of *n* *positive* real numbers equals:

$$\sqrt[n]{\prod_{i=1}^{n} x_i}$$

Key property: Partial substitutability - compensates up to a point/rewards balanced performance/penalises low performance in any of the elements to be aggregated

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Geometric mean

Basic Needs indicators - Country *i*	value
% undernourished people	8
% people with safe drinking water	79
% people with safe sanitation	17

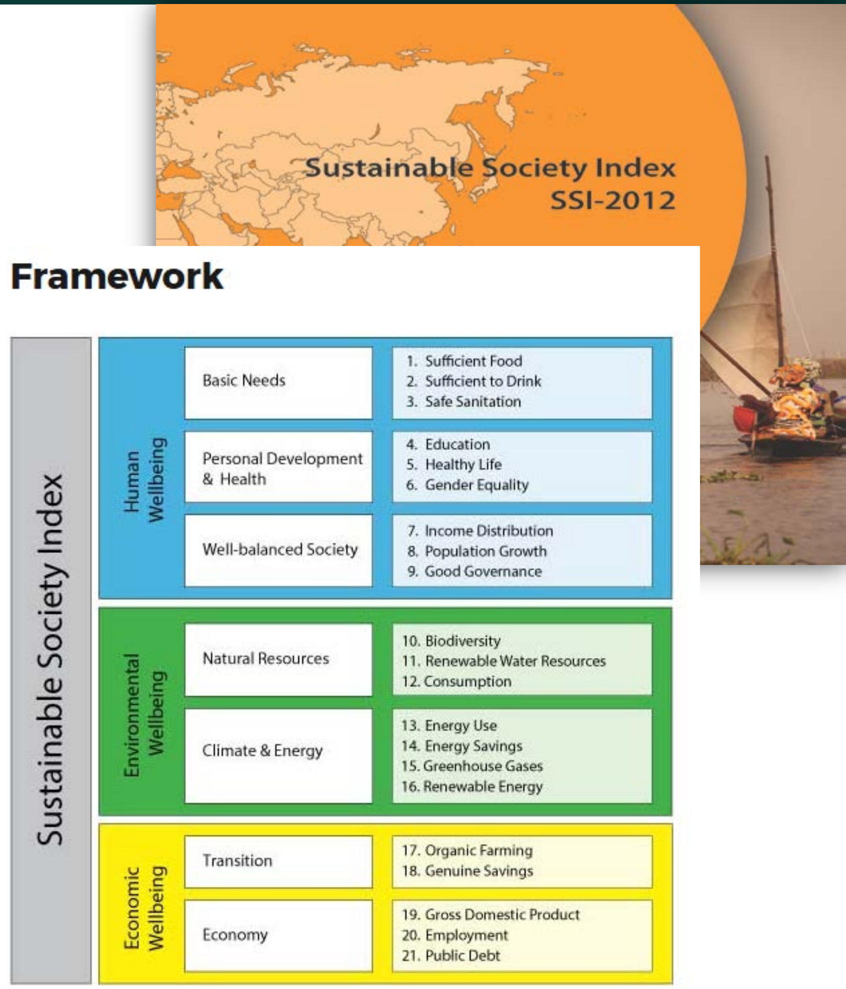
← *Reverse direction*

$$\text{Sufficient Food} = \frac{100-8}{10} = 9.2$$

$$\text{Sufficient to Drink} = \frac{79}{10} = 7.9$$

$$\text{Safe Sanitation} = \frac{17}{10} = 1.7$$

$$\text{Basic Needs} = \sqrt[3]{9.2 \cdot 7.9 \cdot 1.7} = 4.98$$



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Comparing arithmetic vs. geometric mean

	Sufficient Food	Sufficient to Drink		Safe Sanitation		Basic Needs (arithmetic)	Country *i's* improvement	Basic Needs (geometric)	Country *i's* improvement
Country *i* (t)	10.0	8.6		1.4		6.7		4.9	
[1] Country *i* (t+1)	10.0	**9.6**	**+1**	1.4		7.0	4.5%	5.1	4.1%
[2] Country *i* (t+1)	10.0	8.6		**2.4**	**+1**	7.0	4.5%	5.9	20.4%
[3] Country *i* (t+1)	10.0	**9.6**	**+1**	**0.4**	**-1**	6.7	0%	3.4	-31.1%
[4] Country *i* (t+1)	10.0	**7.6**	**-1**	**2.4**	**+1**	6.7	0%	5.7	15.7%

Advantages of using geometric mean vs. arithmetic mean for aggregating scores in the SSI framework:

1) implies only ***partial compensability***, i.e. ***poor performance in one*** element of the framework ***cannot be fully compensated by good performance in another***;
2) ***rewards balance*** by ***penalizing uneven performance*** between dimensions;
3) ***encourages (policy) improvements in the weak dimensions***, i.e. the lower the performance in a particular SSI dimension, the more urgent it becomes to improve in that dimension.

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And what about weights?

- **Weighted linear aggregation**

For a sequence of positive weights w_i, with $\Sigma w_i = 1$, the ***weighted arithmetic mean*** equals:

$$\sum_{i=1}^{n} w_i x_i$$

- **Weighted geometric aggregation**

For a sequence of positive weights w_i, with $\Sigma w_i = 1$, the ***weighted geometric mean*** equals:

$$\prod_{i=1}^{n} {x_i}^{w_i}$$

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Hybrid aggregations

Mixed approach: to create CIs using more than one aggregation functions at different levels of aggregation

Human Development Index (UN)	Arithmetic average within dimension (education)	Geometric average across dimensions
Food and Nutrition Security Index (UN - FAO)	Arithmetic average within dimensions	Geometric average across dimensions

Summary of additive and multiplicative agg.

Common features:

- Normalisation of indicators is required
- Sensitive to outliers (i.e. outlier treatment needed)
- Interval level information is kept in the output (scores, not ranks)
- Weights have the meaning of *trade-offs* (and not of importance coefficients): deficits in one dimension can be offset by a sufficient surplus in another

Differences:

- Implies perfect (constant) substitutability (arith.) vs. partial compensability (geom.) (penalise unbalanced perfomance)
- Arithmetic means are always greater than or equal to equivalent geometric means

And what if quantitative and qualitative criteria are pooled together in the dataset?

Example: Multi-criteria performance matrix (quantitative/qualitative variables as constituent elements of the same conceptual framework)

	Criterion 1 (/20)	Criterion 2 (rating)	Criterion 3 (qual.)	Criterion 4 (Y/N)	…
Action 1	20	135	G	Yes	…
Action 2	9	156	B	Yes	…
Action 3	15	129	VG	No	…
Action 4	9	146	VB	No	…
Action 5	7	121	G	Yes	…
…	…	…	…	…	…

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A quick look behind: from Social Choice Theory to Multi-Criteria Analysis

The problem of Social Choice:

1	Ramon Llull
2	Nicolas de Condorcet
3	Nicholas of Kues
4	Jean-Charles, Chevalier de Borda

- A group of voters has to select a candidate among a group of candidates (election)
- Each voter has a personal ranking of the candidates according to his/her preferences
- Which candidate must be elected?

Best interest of society

What is the «best» voting procedure?

Analogy with Multi-Criteria Analysis:

- Candidates ↔ Alternatives
- Voters ↔ Criteria

Methods based on ranks

Obtain a complete ranking under each criteria (= for each voter) as a starting point

Method #1 : Relative majority

Rank	Points
1	1
2	0
3	0
…	…
N-1	0
N	0

<u>3 candidates:</u> **A**dam, **B**rian, **C**arlos

30 voters:

11 voters	**10** voters	**9** voters
A	B	C
B	C	B
C	A	A

19 voters out of 30 rank A as their least preferred option (strong opposition)

Problem: B and C preferred to A by a majority of voters!

A	11
B	10
C	9

Adam is elected

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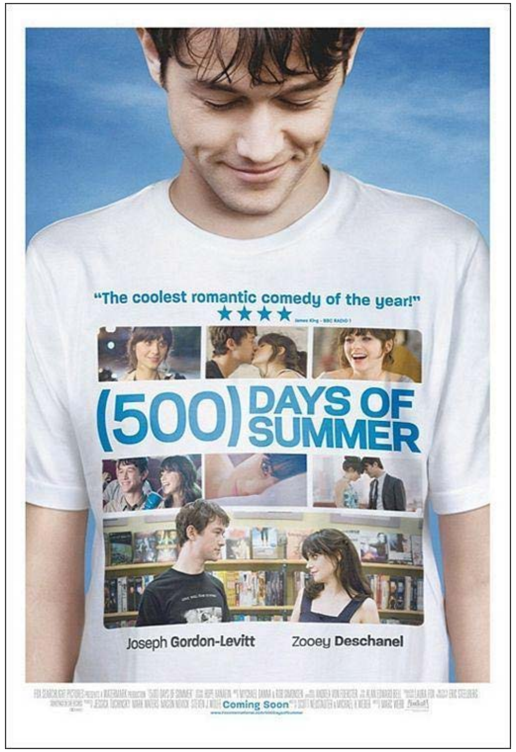
Pit stop #1: You are simply the best!?

Let's have a look at the *Summer Finn* example:



I like this: "I love her smile. I love her hair. I love her knees. I love how she licks her lips before she talks. I love her heart-shaped birthmark on her neck. I love it when she sleeps."

I dislike this: "I hate her crooked teeth. I hate her 1960s haircut. I hate her knobby knees. I hate her cockroach-shaped splotch on her neck. I hate the way she smacks her lips before she talks. I hate the way she sounds when she laughs."

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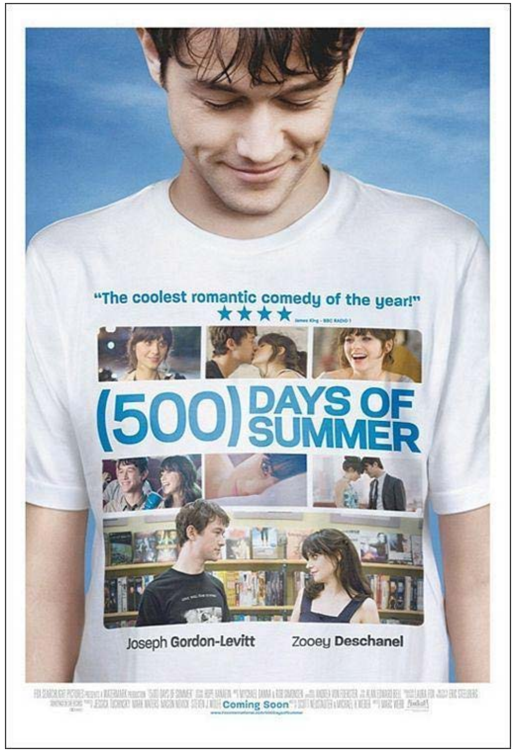
Pit stop #1: You are simply the best!?

Take away lessons from the *Summer Finn* example:



- Question yourself ***not only about*** what are ***the most*** salient (***positive***) ***aspects in the option*** under consideration, but ***check the whole range of criteria*** to help you make a more ***balanced assessent*** (***bring*** also the ***negative aspects/drawbacks to the foreground***)
- What looks like ***the best option*** at first sight :) ***… might*** turn out to ***be the worst all things*** (criteria) ***considered*** :(

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Method #2: Borda

Rank	Borda points
1	N-1
2	N-2
3	N-3
…	…
N-1	1
N	0

3 candidates: **A**dam, **B**rian, **C**arlos

81 voters:

30 voters	**29** voters	**10** voters	**10** voters	**1** voter	**1** voter
A	C	C	B	A	B
C	A	B	A	B	C
B	B	A	C	C	A

Points	**Scores**	
2	A	**101**
1	B	**33**
0	C	**109**

31 x 2 + 39 x 1

11 x 2 + 11 x 1

39 x 2 + 31 x 1

Carlos is elected!

Method #2: Borda

Rank	Borda points
1	N-1
2	N-2
3	N-3
…	…
N-1	1
N	0

4 candidates: **A**dam, **B**rian, **C**arlos, **D**avid

7 voters:

3 voters	**2** voters	**2** voters	Points
C	B	A	3
B	A	D	2
A	D	C	1
D	C	B	0

Scores	
A	13
B	12
C	11
D	6

Ranking
A
B
C
D

Adam is elected

Method #2: Borda

Rank	Borda points
1	N-1
2	N-2
3	N-3
…	…
N-1	1
N	0

4 candidates: **A**dam, **B**rian, **C**arlos, ~~**D**avid~~

7 voters:

3 voters	**2** voters	**2** voters	Points
C	B	A	2
B	A	C	1
A	C	B	0

Just by dropping the last in the ranking, the order of preference for the better ranked alternatives will change

Problem: Fully dependant on irrelevant alternatives (easy to manipulate)

Scores	
A	6
B	7
C	8

Ranking
C
B
A

Carlos is elected

Method #3: Median ranking

<u>3 candidates:</u> **A**dam, **B**rian, **C**arlos

11 voters:

Rank candidates according to each voter (criteria) and then calculate median rank for each candidate <u>across voters</u>

6 voters	**4** voters	**1** voters
A	C	C
C	A	B
B	B	A

A: 11111**1**22223
B:23333**3**33333
C:11111**2**22222

Ranking
A
C
B

Methods based on pairwise comparisons

Confront alternative *i* vs. alternative *j* using the original quantitative/qualitative values for each criteria as a starting point

Method #4: Condorcet

3 candidates: **A**dam**, B**rian**, C**arlos

30 voters:

11 voters	**10** voters	**9** voters
A	B	C
B	C	B
C	A	A

Search for a **Condorcet winner**, i.e. an alternative preferred over every other in pairwise comparisons

B preferred to **A**	19 votes
B preferred to **C**	21 votes
C preferred to **A**	19 votes

Brian is elected

Method #4: Condorcet

3 candidates: **A**dam, **B**rian, **C**arlos

9 voters:

4 voters	**3** voters	**2** voters
A	B	C
B	C	A
C	A	B

Problem: The Condorcet winner might not exist – nobody is elected! (cycle)

A preferred to **B**	6 votes
B preferred to **C**	7 votes
C preferred to **A**	5 votes

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Pit stop #2: Why is the presence of voting cycles so important in the agg. and ordering of preferences?

The *Tuesday movie night* manipulative dilemma!

Three friends:

Marcos (ES – *the host*)

Marco (IT)

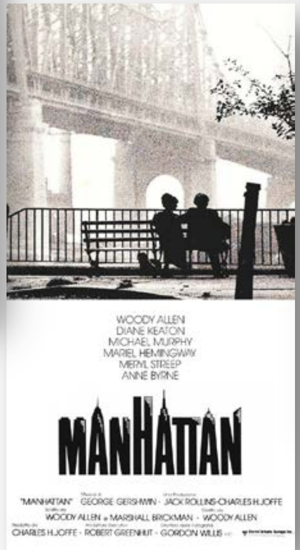
Marko (HR)

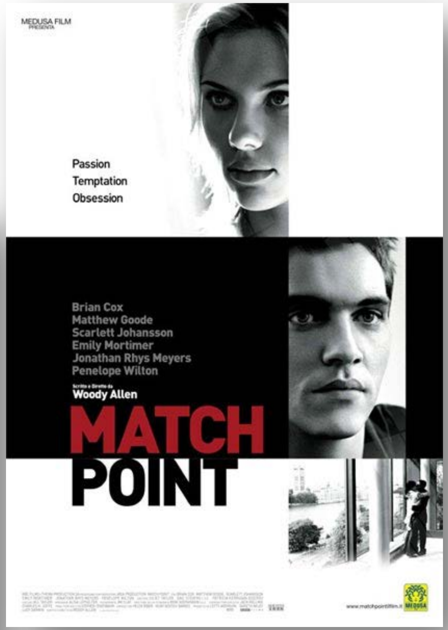
Three movies:

Melinda and Melinda (M1_Marcos'#1)

Manhattan (M2_Marco's#1)

Match Point (M3_Marko's#1)

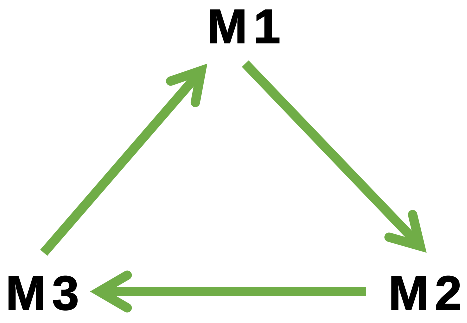




Pit stop #2: Why is the presence of voting cycles so important in the agg. and ordering of preferences?

Working out the *Tuesday movie night* manipulative dilemma when you know the preferences of your guests:

Complete ranking of preferences:		
Marcos	**Marco**	**Marko**
M1	M2	M3
M2	M3	M1
M3	M1	M2



Vote graph for the Tuesday movie night session

Let's vote again but start with a different one to start with:

M2 vs M3 → M2 (Mcs & Mc vs Mk)

M2 vs M1 → M1 (Mcs & Mk vs Mc)

So tonight we watch: M1! (***Marcos is the happiest one in the room!***)

And what if we start with M3?:

M1 vs M3 → M3 (Mc & Mk vs Mcs)

M3 vs M2 → M2 (Mcs & Mc vs Mk)

So tonight we watch: M2! (***Marko is the happiest one in the room!***)

So eventually the host (Marco, a middle name *Machiavelli*) might consider ***manouvering to secure that his preferred option*** (M1 - Melinda & Melinda) ***will be the one selected***, by just proposing to start the vote with Manhattan (M2) vs. Match Point (M3)!!!

And what if the dataset is *quantitative… but highly heterogeneous*?

Your data might be strictly quantitative but plagued with ***poor or*** even ***negative correlations*** among indicators… which links to the problem of Social Choice, where preferences of voters are not expected to be related (voters might hold independent and even opposite views which still need to be aggregated)

You should not go ahead using standard aggregation (*averaging*) methods, because results would then be <u>***highly sensitive***</u> to underlying methodological and conceptual choices:

- conceptual grouping of indicators into themes
- missing data estimation method (or none…)
- data treatment (of highly skewed variables)
- data normalisation
- weights
- aggregation formula

The ESRB Heatmap dataset

			average	st.d.	min	max	skew	kurt
MACRO	Current real GDP growth	2.1	0.10	2.60	-7.92	5.99	-0.12	1.07
MACRO	Domestic credit-to-GDP gap	2.2	-6.77	6.06	-21.88	0.41	-1.11	0.30
MACRO	Current account balance-to-GDP ratio	2.3	0.02	4.09	-9.89	10.27	0.65	0.18
MACRO	Rate of unemployment	2.4	10.83	5.17	4.15	26.22	1.35	1.83
FISCAL	Forecast government debt-to-GDP ratio	2.5	67.83	35.26	6.25	170.32	0.65	0.30
FISCAL	Forecast government deficit-to-GDP ratio	2.6	4.02	2.83	0.15	13.38	0.93	0.91
FISCAL	Credit default swap premia on sovereign debt	2.7	574.16	1836.22	18.63	12447.07	**5.57**	**31.71**
FISCAL	Annual sovereign debt redemptions as a share of GDP	2.8	14.51	11.07	0.00	47.37	0.81	-0.09
HH	Households' debt-to-gross disposable income ratio	2.9	104.67	61.58	36.88	268.92	1.23	0.86
HH	Estimates of the over/undervaluation of residential property prices	3.1.a.	2.58	11.85	-12.67	28.39	0.62	-0.93
HH	Share of foreign currency loans on total loans to non-MFIs	3.2a	18.81	25.58	0.28	89.45	1.55	0.97
HH	MFI lending to HH (annual growth rates) NEW	n.a.2	0.66	4.75	-16.69	11.12	-0.68	1.92
NFC	Non-financial corporations' debt-to-GDP ratio	2.13	115.88	74.87	0.00	555.04	**2.74**	**14.13**
NFC	MFI lending to NFC (annual growth rates) NEW	n.a.1	0.50	5.02	-10.79	14.01	0.23	-0.41
MFIs	Share of central bank funding in credit institutions liabilities	4.5	4.60	7.28	0.00	34.78	**2.58**	**7.24**
MFIs	MFI's exposure to domestic sovereign (share of total assets) NEW	n.a.3	0.08	0.06	0.00	0.23	0.92	-0.11
MFIs	Banking sector leverage NEW	n.a.4	16.16	7.22	4.98	50.46	1.35	**4.87**
MFIs	Loan to deposit ratio NEW	n.a.5	1.31	0.47	0.61	2.97	1.93	**4.43**

Notes: raw data, pooled dataset: 2013Q3, 2012Q4, 2012 Q3, 2011 Q4 (four time-points x 28 countries)

Outliers in three indicators (problematic when analyzing the correlation structure)

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The ESRB Heatmap dataset

			average	st.d.	min	max	skew	kurt
MACRO	Current real GDP growth	2.1	0.10	2.60	-7.92	5.99	-0.12	1.07
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	MFI's exposure to domestic sovereign (share of total assets) NEW	n.a.3	0.08	0.06	0.00	0.23	0.92	-0.11
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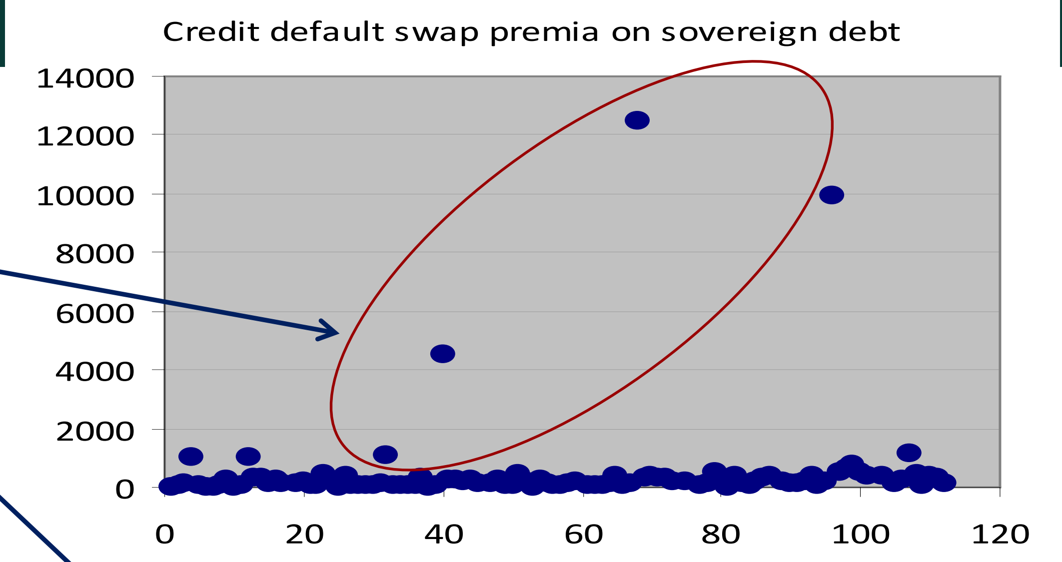
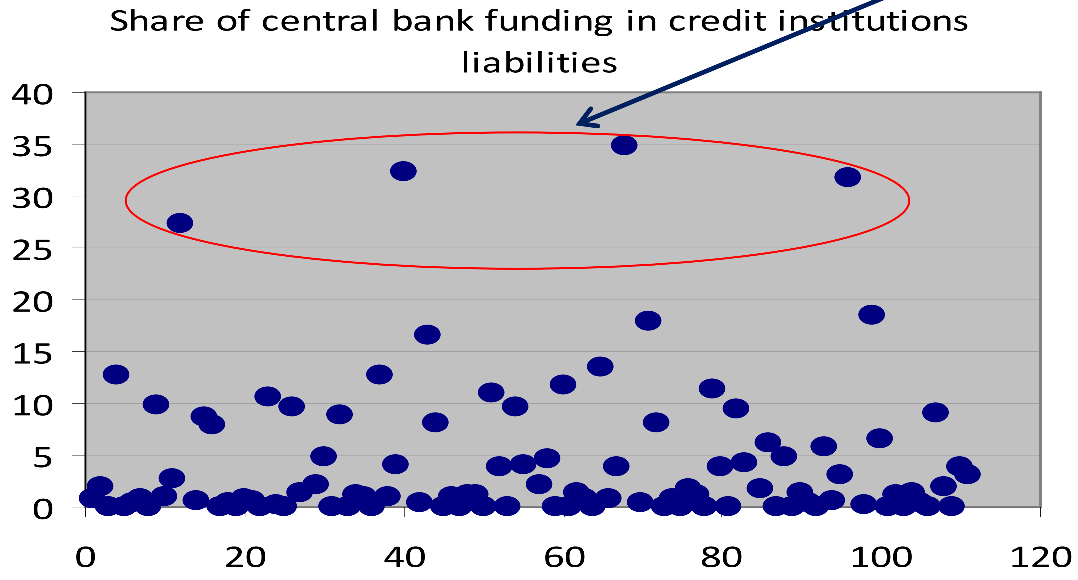
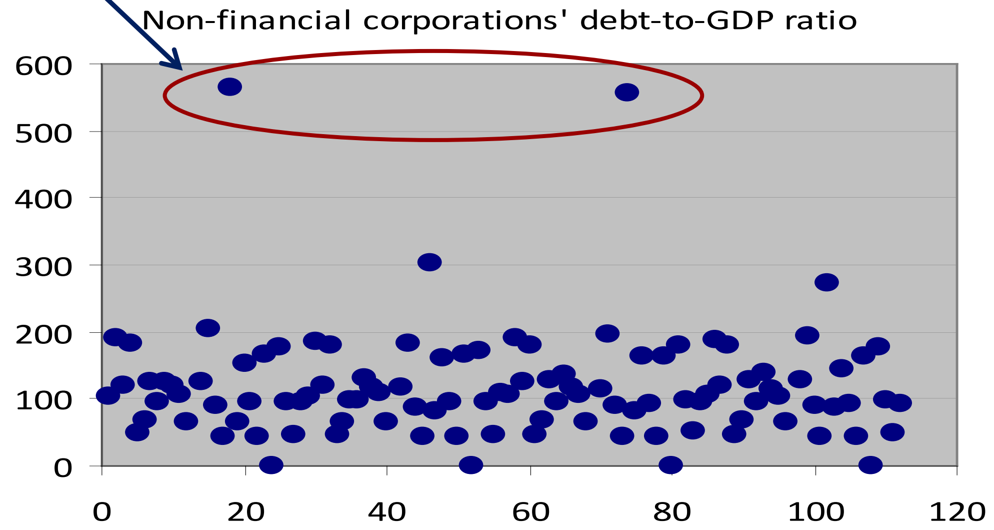




Need to be treated before the correlation analysis and calculation of an aggregate score based on linear/geometric aggregations

Shouldn't be treated because of conceptual reasons (extreme behaviour needs to be accounted for)



The ESRB Heatmap dataset

Correlation structure in the ESRB country heat map

	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.1.a.	3.2a	n.a.2	2.13	n.a.1	4.5	n.a.3	n.a.4	n.a.5
2.1	[1 ,1]	[-0.2 ,0.1]	[0.1 ,0.2]	[-0.4 ,-0.2]	[-0.7 ,-0.6]	[-0.6 ,-0.2]	[-0.7 ,-0.5]	[-0.6 ,-0.4]	[-0.4 ,-0.2]	[-0.3 ,0.3]	[0.4 ,0.6]	[0 ,0.2]	[-0.3 ,-0.1]	[0 ,0.4]	[-0.7 ,-0.5]	[-0.2 ,-0.1]	[-0.6 ,-0.4]	[-0.2 ,-0.1]
2.2	[-0.2 ,0.1]	[1 ,1]	[-0.3 ,0]	[-0.6 ,-0.1]	[-0.2 ,0.1]	[-0.5 ,-0.3]	[-0.1 ,0]	[0 ,0.3]	[-0.5 ,-0.5]	[0.1 ,0.5]	[-0.1 ,0]	[0.2 ,0.5]	[-0.5 ,-0.4]	[0.5 ,0.6]	[-0.5 ,-0.1]	[0.3 ,0.6]	[-0.2 ,-0.1]	[-0.4 ,-0.3]
2.3	[0.1 ,0.2]	[-0.3 ,0]	[1 ,1]	[-0.6 ,-0.2]	[-0.4 ,-0.2]	[-0.5 ,-0.3]	[-0.6 ,-0.4]	[-0.2 ,0]	[0.4 ,0.5]	[-0.4 ,-0.3]	[-0.3 ,-0.2]	[0.1 ,0.1]	[0.2 ,0.4]	[-0.1 ,-0.1]	[-0.4 ,-0.2]	[-0.5 ,-0.2]	[0.1 ,0.4]	[0.3 ,0.4]
2.4	[-0.4 ,-0.2]	[-0.6 ,-0.1]	[-0.6 ,-0.2]	[1 ,1]	[0.2 ,0.5]	[0.6 ,0.7]	[0.1 ,0.8]	[0 ,0.2]	[-0.1 ,0.1]	[-0.1 ,0.1]	[0 ,0.2]	[-0.4 ,-0.1]	[-0.1 ,0.1]	[-0.4 ,-0.3]	[0.3 ,0.7]	[0.1 ,0.3]	[-0.2 ,0.3]	[0 ,0.1]
2.5	[-0.7 ,-0.6]	[-0.2 ,0.1]	[-0.4 ,-0.2]	[0.2 ,0.5]	[1 ,1]	[0.5 ,0.7]	[0.6 ,0.6]	[0.6 ,0.7]	[0.1 ,0.2]	[-0.5 ,0.2]	[-0.4 ,-0.3]	[-0.4 ,-0.3]	[0 ,0.1]	[-0.5 ,-0.2]	[0.8 ,0.8]	[0 ,0.1]	[0.4 ,0.6]	[-0.1 ,-0.1]
2.6	[-0.6 ,-0.2]	[-0.5 ,-0.3]	[-0.5 ,-0.3]	[0.6 ,0.7]	[0.5 ,0.7]	[1 ,1]	[0.4 ,0.6]	[0.2 ,0.4]	[0.1 ,0.2]	[-0.3 ,0.1]	[-0.3 ,0]	[-0.3 ,-0.2]	[-0.1 ,0.1]	[-0.5 ,-0.2]	[0.6 ,0.8]	[0.1 ,0.2]	[-0.1 ,0.4]	[-0.1 ,0.1]
2.7	[-0.7 ,-0.5]	[-0.1 ,0]	[-0.6 ,-0.4]	[0.1 ,0.8]	[0.6 ,0.6]	[0.4 ,0.6]	[1 ,1]	[0.2 ,0.6]	[-0.2 ,0.1]	[-0.4 ,0.2]	[-0.1 ,0.1]	[-0.5 ,-0.3]	[0 ,0.2]	[-0.6 ,-0.3]	[0.7 ,0.8]	[0 ,0.2]	[-0.2 ,0.5]	[-0.2 ,-0.1]
2.8	[-0.6 ,-0.4]	[0 ,0.3]	[-0.2 ,0]	[0 ,0.2]	[0.6 ,0.7]	[0.2 ,0.4]	[0.2 ,0.6]	[1 ,1]	[-0.1 ,0.3]	[-0.2 ,0.5]	[-0.4 ,-0.3]	[-0.3 ,0]	[0.1 ,0.2]	[-0.2 ,0.1]	[0.4 ,0.4]	[0 ,0.3]	[0.2 ,0.6]	[-0.2 ,-0.1]
2.9	[-0.4 ,-0.2]	[-0.5 ,-0.5]	[0.4 ,0.5]	[-0.1 ,0.1]	[0.1 ,0.2]	[0.1 ,0.2]	[-0.2 ,0.1]	[-0.1 ,0.3]	[1 ,1]	[-0.5 ,-0.1]	[-0.4 ,-0.3]	[-0.1 ,0.1]	[0.5 ,0.5]	[-0.2 ,0]	[0.1 ,0.2]	[-0.6 ,-0.5]	[0.4 ,0.6]	[0.5 ,0.6]
3.1.a.	[-0.3 ,0.3]	[0.1 ,0.5]	[-0.4 ,-0.3]	[-0.1 ,0.1]	[-0.5 ,0.2]	[-0.3 ,0.1]	[-0.4 ,0.2]	[-0.2 ,0.5]	[-0.5 ,-0.1]	[1 ,1]	[-0.4 ,0]	[0 ,0.7]	[0 ,0.3]	[0.2 ,0.4]	[-0.4 ,0.3]	[-0.1 ,0.1]	[0.2 ,0.4]	[-0.2 ,0.3]
3.2a	[0.4 ,0.6]	[-0.1 ,0]	[-0.3 ,-0.2]	[0 ,0.2]	[-0.4 ,-0.3]	[-0.3 ,0]	[-0.1 ,0.1]	[-0.4 ,-0.3]	[-0.4 ,-0.3]	[-0.4 ,0]	[1 ,1]	[-0.4 ,-0.4]	[-0.4 ,-0.3]	[0 ,0.3]	[-0.3 ,-0.3]	[0.1 ,0.1]	[-0.5 ,-0.4]	[-0.2 ,-0.1]
n.a.2	[0 ,0.2]	[0.2 ,0.5]	[0.1 ,0.1]	[-0.4 ,-0.1]	[-0.4 ,-0.3]	[-0.3 ,-0.2]	[-0.5 ,-0.3]	[-0.3 ,0]	[-0.1 ,0.1]	[0 ,0.7]	[-0.4 ,-0.4]	[1 ,1]	[-0.1 ,0.1]	[0.2 ,0.6]	[-0.5 ,-0.2]	[0 ,0.1]	[-0.1 ,0]	[-0.1 ,0]
2.13	[-0.3 ,-0.1]	[-0.5 ,-0.4]	[0.2 ,0.4]	[-0.1 ,0.1]	[0 ,0.1]	[-0.1 ,0.1]	[0 ,0.2]	[0.1 ,0.2]	[0.5 ,0.5]	[0 ,0.3]	[-0.4 ,-0.3]	[-0.1 ,0.1]	[1 ,1]	[-0.4 ,-0.2]	[0.1 ,0.2]	[-0.6 ,-0.6]	[0.1 ,0.4]	[0.1 ,0.2]
n.a.1	[0 ,0.4]	[0.5 ,0.6]	[-0.1 ,-0.1]	[-0.4 ,-0.3]	[-0.5 ,-0.2]	[-0.5 ,-0.2]	[-0.6 ,-0.3]	[-0.2 ,0.1]	[-0.2 ,0]	[0.2 ,0.4]	[0 ,0.3]	[0.2 ,0.6]	[-0.4 ,-0.2]	[1 ,1]	[-0.6 ,-0.3]	[0 ,0.4]	[-0.3 ,0.1]	[-0.3 ,0]
4.5	[-0.7 ,-0.5]	[-0.5 ,-0.1]	[-0.4 ,-0.2]	[0.3 ,0.7]	[0.8 ,0.8]	[0.6 ,0.8]	[0.7 ,0.8]	[0.4 ,0.4]	[0.1 ,0.2]	[-0.4 ,0.3]	[-0.3 ,-0.3]	[-0.5 ,-0.2]	[0.1 ,0.2]	[-0.6 ,-0.3]	[1 ,1]	[0 ,0.1]	[0.2 ,0.5]	[0 ,0.1]
n.a.3	[-0.2 ,-0.1]	[0.3 ,0.6]	[-0.5 ,-0.2]	[0.1 ,0.3]	[0 ,0.1]	[0.1 ,0.2]	[0 ,0.2]	[0 ,0.3]	[-0.6 ,-0.5]	[-0.1 ,0.1]	[0.1 ,0.1]	[0 ,0.1]	[-0.6 ,-0.6]	[0 ,0.4]	[0 ,0.1]	[1 ,1]	[-0.4 ,-0.3]	[-0.3 ,-0.3]
n.a.4	[-0.6 ,-0.4]	[-0.2 ,-0.1]	[0.1 ,0.4]	[-0.2 ,0.3]	[0.4 ,0.6]	[-0.1 ,0.4]	[-0.2 ,0.5]	[0.2 ,0.6]	[0.4 ,0.6]	[0.2 ,0.4]	[-0.5 ,-0.4]	[-0.1 ,0]	[0.1 ,0.4]	[-0.3 ,0.1]	[0.2 ,0.5]	[-0.4 ,-0.3]	[1 ,1]	[0.2 ,0.4]
n.a.5	[-0.2 ,-0.1]	[-0.4 ,-0.3]	[0.3 ,0.4]	[0 ,0.1]	[-0.1 ,-0.1]	[-0.1 ,0.1]	[-0.2 ,-0.1]	[-0.2 ,-0.1]	[0.5 ,0.6]	[-0.2 ,0.3]	[-0.2 ,-0.1]	[-0.1 ,0]	[0.1 ,0.2]	[-0.3 ,0]	[0 ,0.1]	[-0.3 ,-0.3]	[0.2 ,0.4]	[1 ,1]

Notes: raw data (without outliers), pooled dataset: 2013Q3, 2012Q4, 2012 Q3, 2011 Q4, correlations less than 0.38 are not significant

Weak correlation structure: poor (or negative) correlations / correlation structure changing over time

The ESRB Heatmap dataset

Correlation structure in the ESRB country heat map

Example: Macro dimension

	2.1	2.2	2.3	2.4
2.1	[1 ,1]	[-0.2 ,0.1]	[0.1 ,0.2]	[-0.4 ,-0.2]
2.2	[-0.2 ,0.1]	[1 ,1]	[-0.3 ,0]	[-0.6 ,-0.1]
2.3	[0.1 ,0.2]	[-0.3 ,0]	[1 ,1]	[-0.6 ,-0.2]
2.4	[-0.4 ,-0.2]	[-0.6 ,-0.1]	[-0.6 ,-0.2]	[1 ,1]

Notes: raw data (without outliers), pooled dataset: 2013Q3, 2012Q4, 2012 Q3, 2011 Q4, correlations less than 0.38 are not significant

- Most bivariate correlations are not significant at any of the 4 time-points
- No bivariate correlation is significant at all four time points
- Presence of significantly negative correlations

Method #5: Kemeny order (Condorcet, ***Kemeny***, Young and Levenglick C-K-Y-L)

Features of the Kemeny algorithm (fully non-compensatory approach):

- no impact of outliers;
- no need for data normalisation
- no need for uniform covariance matrix/”good” correlation structure;
- can be used both with continuous and categorical variables;
- no use of any linear or multiplicative functional form;
- ***only weights and orientation*** are ***required***;
- use of the ***weights*** assigned to indicators <u>exactly</u> ***as importance coefficients***;
- works as ***a non-arbitrary cycle breaking rule*** based on ***a maximum likelihood approach***
- satisfies Condorcet criterion and a limited version of the IIA property
- seemingly straightforward, but much higher computational requirements than previous methods!

Sources: Athanasoglou (2015) , Tarjan (1972), Van Zuylen, and Williamson (2009), Munda and Nardo (2009)

As a working example: How could one assess the overall risk in a given country and year within the ESRB framework using this maximum likelihood approach?

Method #5: Kemeny order

Data for 2013 Q3	FISCAL			
	Forecast government debt-to-GDP ratio	Forecast government deficit-to-GDP ratio	Credit default swap premia on sovereign debt	Annual sovereign debt redemptions as a share of GDP
	2.5	2.6	2.7	2.8
Orientation	-1	-1	-1	-1
WEIGHT	0.25	0.25	0.25	0.25
MT	74.9	3.6	205.0	14.7
NL	75.8	3.6	47.9	11.6
PL	58.9	4.1	87.3	5.8
PT	124.3	4.0	445.5	18.8
RO	38.5	2.4	207.6	7.3
SE	39.0	0.4	17.7	7.0
SI	66.5	4.9	350.9	5.6
SK	56.7	3.1	87.8	9.2

Step 1 – Raw data & Weights & Orientation

Method #5: Kemeny order

Data for 2013 Q3	FISCAL			
	Forecast government debt-to-GDP ratio	Forecast government deficit-to-GDP ratio	Credit default swap premia on sovereign debt	Annual sovereign debt redemptions as a share of GDP
	2.5	2.6	2.7	2.8
Orientation	-1	-1	-1	-1
WEIGHT	0.25	0.25	0.25	0.25
MT	74.9	3.6	205.0	14.7
NL	75.8	3.6	47.9	11.6
PL	58.9	4.1	87.3	5.8
PT	124.3	4.0	445.5	18.8
RO	38.5	2.4	207.6	7.3
SE	39.0	0.4	17.7	7.0
SI	66.5	4.9	350.9	5.6
SK	56.7	3.1	87.8	9.2
		3.62		
		3.56		

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

For each pairwise comparison, the weights for the indicators that favour A versus B are added up = concordance index. In case of ties, weights are split between countries. **For *n* countries, there are *n (n-1)* pairwise comparisons to be made**

Example:

MT versus NL = 0.25

NL versus MT = 0.75

Sum = 1.00

European Commission

Method #5: Kemeny order

Data for 2013 Q3	FISCAL			
	Forecast government debt-to-GDP ratio	Forecast government deficit-to-GDP ratio	Credit default swap premia on sovereign debt	Annual sovereign debt redemptions as a share of GDP
	2.5	2.6	2.7	2.8
Orientation	-1	-1	-1	-1
WEIGHT	0.25	0.25	0.25	0.25
MT	74.9	3.6	205.0	14.7
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SE	39.0	0.4	17.7	7.0
SI	66.5	4.9	350.9	5.6
SK	56.7	3.1	87.8	9.2

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

For each pairwise comparison, the weights for the indicators that favour A versus B are added up = concordance index. In case of ties, weights are split between countries. **For *n* countries, there are *n (n-1)* pairwise comparisons to be made**

Example:

MT versus PT = 1.00

PT versus MT = 0.00

Sum = 1.00

Method #5: Kemeny order

Outranking matrix

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

Step 3 – Outranking matrix

All concordance values are entered in a so called outranking matrix. (entries above and below the diagonal sum up to 1.0)

Method #5: Kemeny order

Outranking matrix

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

Step 3 – Outranking matrix

Step 4 – Maximum Likelihood ranking (highest support score)

Of all possible permutations of rankings, find the one that ***maximises the sum of elements above the diagonal*** (= ***maximise total pairwise support = minimise total disagreement/pairwise inversions***)

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Method #5: Kemeny order

Outranking matrix

	SE	RO	PL	SK	SI	NL	MT	PT
SE	0.00	0.75	0.75	1.00	0.75	1.00	1.00	1.00
RO	0.25	0.00	0.50	0.75	0.75	0.75	0.75	1.00
PL	0.25	0.50	0.00	0.50	0.75	0.50	0.75	0.75
SK	0.00	0.25	0.50	0.00	0.75	0.75	1.00	1.00
SI	0.25	0.25	0.25	0.25	0.00	0.50	0.50	0.75
NL	0.00	0.25	0.50	0.25	0.50	0.00	0.75	1.00
MT	0.00	0.25	0.25	0.00	0.50	0.25	0.00	1.00
PT	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

Step 3 – Outranking matrix

Step 4 – Maximum Likelihood ranking (highest support score)

	Rank
SE	1
RO	2
PL	3
SK	4
SI	5
NL	6
MT	7
PT	8

Note: A Kemeny order is not necessarily unique

Method #5: Kemeny order

"The more important issue is whether the [C-K-Y-L] method is *intuitively easy to grasp*, and whether it *improves on methods currently in use*. On both these counts I think that the answer is affirmative, and <u>I predict</u> that the <u>time will come</u> when it is considered a <u>standard tool for political and group decision making</u>."

Peyton Young
Professor Emeritus, Research Professor in Economics, Johns Hopkins University

Quick-searching algorithms

- Given the of computational requirements and the risk of factorial permutation overload when it comes to implementing the C-K-Y-L algorithm (NP-hard to compute), quick-searching algorithms have been developed to approximate the optimal solution and to arrive at a compromise ranking

- **Arrow-Raynaud** and **Copeland** are two of the most popular quick-searching algorithms

Method #6: Arrow-Raynaud algorithm

Outranking matrix

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

Min of max

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

Step 3 – Outranking matrix

Step 4 – Arrow-Raynaud algorithm (Kemeny approx.)

a) Identify *eij* max along each row in the outranking matrix.
b) Choose min of the maxima.
c) Delete corresponding row and column. [The row of this minimum corresponds to a country that will be ranked at the (n-r+1) position].
d) Repeat step till the outranking matrix becomes void.

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Method #6: Arrow-Raynaud algorithm

Outranking matrix

Step 1 – Raw data & Weights & Orientation
Step 2 – Concordance index
Step 3 – Outranking matrix
Step 4 – Arrow-Raynaud

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

Min of max

	Rank
MT	
NL	
PL	
PT	8
RO	
SE	
SI	
SK	

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Method #6: Arrow-Raynaud algorithm

Step 1 – Raw data & Weights & Orientation
Step 2 – Concordance index
Step 3 – Outranking matrix
Step 4 – Arrow-Raynaud

Outranking matrix

	MT	NL	PL	RO	SE	SI	SK
MT	0.00	0.25	0.25	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.50	0.25	0.75	0.50
RO	0.75	0.75	0.50	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	0.25	0.00	0.75	0.00

Min of max

	Rank
MT	7
NL	
PL	
PT	8
RO	
SE	
SI	7
SK	

Min of max

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Method #6: Arrow-Raynaud algorithm

Step 1 – Raw data & Weights & Orientation
Step 2 – Concordance index
Step 3 – Outranking matrix
Step 4 – Arrow-Raynaud

Outranking matrix

	NL	PL	RO	SE	SI	SK
NL	0.00	0.50	0.25	0.00	0.50	0.25
PL	0.50	0.00	0.50	0.25	0.75	0.50
RO	0.75	0.50	0.00	0.25	0.75	0.75
SE	1.00	0.75	0.75	0.00	0.75	1.00
SI	0.50	0.25	0.25	0.25	0.00	0.25
SK	0.75	0.50	0.25	0.00	0.75	0.00

Min of max

	Rank
MT	7
NL	6
PL	
PT	8
RO	
SE	
SI	
SK	

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Method #6: Arrow-Raynaud algorithm

Step 1 – Raw data & Weights & Orientation
Step 2 – Concordance index
Step 3 – Outranking matrix
Step 4 – Arrow-Raynaud

Outranking matrix

	PL	RO	SE	SI	SK
PL	0.00	0.50	0.25	0.75	0.50
RO	0.50	0.00	0.25	0.75	0.75
SE	0.75	0.75	0.00	0.75	1.00
SI	0.25	0.25	0.25	0.00	0.25
SK	0.50	0.25	0.00	0.75	0.00

Min of max

	Rank
MT	7
NL	6
PL	
PT	8
RO	
SE	
SI	5
SK	

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Method #6: Arrow-Raynaud algorithm

Step 1 – Raw data & Weights & Orientation
Step 2 – Concordance index
Step 3 – Outranking matrix
Step 4 – Arrow-Raynaud

Outranking matrix

	PL	RO	SE	SK
PL	0.00	0.50	0.25	0.50
RO	0.50	0.00	0.25	0.75
SE	0.75	0.75	0.00	1.00
SK	0.50	0.25	0.00	0.00

Min of max

	Rank
MT	7
NL	6
PL	
PT	8
RO	
SE	
SI	5
SK	4

Method #6: Arrow-Raynaud algorithm

Step 1 – Raw data & Weights & Orientation
Step 2 – Concordance index
Step 3 – Outranking matrix
Step 4 – Arrow-Raynaud

Outranking matrix

	PL	RO	SE
PL	0.00	0.50	0.25
RO	0.50	0.00	0.25
SE	0.75	0.75	0.00

Min of max

	Rank
MT	7
NL	6
PL	3
PT	8
RO	
SE	
SI	5
SK	4

Method #6: Arrow-Raynaud algorithm

Step 1 – Raw data & Weights & Orientation
Step 2 – Concordance index
Step 3 – Outranking matrix
Step 4 – Arrow-Raynaud

Outranking matrix

	RO	SE
RO	0.00	0.25
SE	0.75	0.00

Min of max

	Rank
MT	7
NL	6
PL	3
PT	8
RO	2
SE	1
SI	5
SK	4

Method #6: Arrow-Raynaud algorithm

Step 1 – Raw data & Weights & Orientation
Step 2 – Concordance index
Step 3 – Outranking matrix
Step 4 – Arrow-Raynaud

Outranking matrix

	SE	RO	PL	SK	SI	NL	MT	PT
SE	0.00	0.75	0.75	1.00	0.75	1.00	1.00	1.00
RO	0.25	0.00	0.50	0.75	0.75	0.75	0.75	1.00
PL	0.25	0.50	0.00	0.50	0.75	0.50	0.75	0.75
SK	0.00	0.25	0.50	0.00	0.75	0.75	1.00	1.00
SI	0.25	0.25	0.25	0.25	0.00	0.50	0.50	0.75
NL	0.00	0.25	0.50	0.25	0.50	0.00	0.75	1.00
MT	0.00	0.25	0.25	0.00	0.50	0.25	0.00	1.00
PT	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00

	Rank
MT	7
NL	6
PL	3
PT	8
RO	2
SE	1
SI	5
SK	4

Method #6: Arrow-Raynaud algorithm

- AR belongs to the class of *prudent ranking rules* (Lamboray, 2007; 2009).
- Others prudent rules include: Kohler's rule (Kohler, 1978) and Ranked Pairs (Tideman, 1987; Zavist and Tideman, 1989).
- Prudent ranking rules select rankings that resolve circular ambiguities (Condorcet cycles) in a way that minimizes the *maximum* pairwise disagreement across all pairs of alternatives. The set of such prudent rankings can be very LARGE.

	SE	RO	PL	SK	SI	NL	MT	PT
SE	0.00	0.75	0.75	1.00	0.75	1.00	1.00	1.00
RO	0.25	0.00	0.50	0.75	0.75	0.75	0.75	1.00
PL	0.25	0.50	0.00	0.50	0.75	0.50	0.75	0.75
SK	0.00	0.25	0.50	0.00	0.75	0.75	1.00	1.00
SI	0.25	0.25	0.25	0.25	0.00	0.50	0.50	0.75
NL	0.00	0.25	0.50	0.25	0.50	0.00	0.75	1.00
MT	0.00	0.25	0.25	0.00	0.50	0.25	0.00	1.00
PT	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00

Just look above the diagonal: minimum is 50%!

Method #6: Arrow-Raynaud algorithm

- AR belongs to the class of *prudent ranking rules* (Lamboray, 2007; 2009).
- Others prudent rules include: Kohler's rule (Kohler, 1978) and Ranked Pairs (Tideman, 1987; Zavist and Tideman, 1989).
- Prudent ranking rules select rankings that resolve circular ambiguities (Condorcet cycles) in a way that minimizes the *maximum* pairwise disagreement across all pairs of alternatives. The set of such prudent rankings can be very LARGE.

11 Equivalent rankings resulting from the AR algorithm

MT	**7**	7	7	7	7	7	7	6	6	6	6
NL	**6**	6	6	5	5	5	4	5	5	4	5
PL	**3**	4	2	4	3	2	5	4	3	5	2
PT	**8**	8	8	8	8	8	8	8	8	8	8
RO	**2**	2	3	2	2	3	2	2	2	2	3
SE	**1**	1	1	1	1	1	1	1	1	1	1
SI	**5**	5	5	6	6	6	6	7	7	7	7
SK	**4**	3	4	3	4	4	3	3	4	3	4

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Method #6: Arrow-Raynaud algorithm

- Prudent rankings do not necessarily coincide with maximum likelihood (ML) ones (Lamboray, 2007). The latter minimize the *total sum* of pairwise disagreements across all pairs of alternatives, not their maximum.
- However… For our purposes, when input data are positively correlated, the highest-likelihood AR ranking will very often (but not always) also be a ML ranking.
- Pathological examples of non-ML prudent rankings in Lamboray (2007) all have input data with high negative correlation.

	SE	RO	PL	SK	SI	NL	MT	PT
SE	0.00	0.75	0.75	1.00	0.75	1.00	1.00	1.00
RO	0.25	0.00	0.50	0.75	0.75	0.75	0.75	1.00
PL	0.25	0.50	0.00	0.50	0.75	0.50	0.75	0.75
SK	0.00	0.25	0.50	0.00	0.75	0.75	1.00	1.00
SI	0.25	0.25	0.25	0.25	0.00	0.50	0.50	0.75
NL	0.00	0.25	0.50	0.25	0.50	0.00	0.75	1.00
MT	0.00	0.25	0.25	0.00	0.50	0.25	0.00	1.00
PT	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00

In fact, for this dataset 2013 Q3, all 11 equivalent AR orderings are also the ML orderings

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Method #7: Copeland rule

If Kemeny is the optimal solution, Copeland is a good approximation with the advantage of “button-click” speed

How does Copeland work? ***Wins*** minus ***Defeats***

Outranking matrix

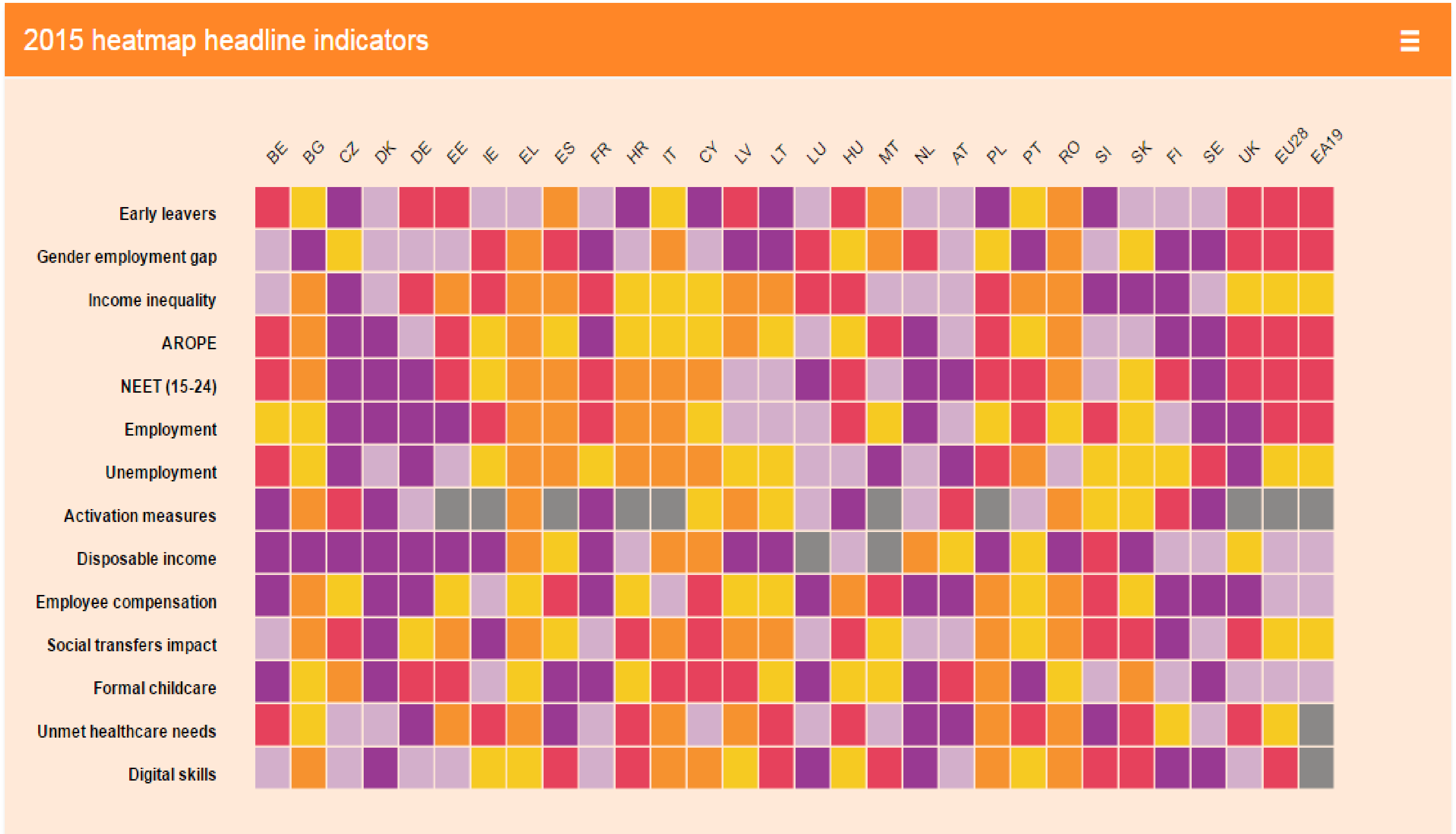
	SE	RO	PL	SK	SI	NL	MT	PT
SE	0.00	0.75	0.75	1.00	0.75	1.00	1.00	1.00
RO	0.25	0.00	0.50	0.75	0.75	0.75	0.75	1.00
PL	0.25	0.50	0.00	0.50	0.75	0.50	0.75	0.75
SK	0.00	0.25	0.50	0.00	0.75	0.75	1.00	1.00
SI	0.25	0.25	0.25	0.25	0.00	0.50	0.50	0.75
NL	0.00	0.25	0.50	0.25	0.50	0.00	0.75	1.00
MT	0.00	0.25	0.25	0.00	0.50	0.25	0.00	1.00
PT	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00

	Wins	Defeats	Total	Rank
SE	7	0	7	1
RO	5	1	4	2
PL	3	1	2	3
SK	4	2	2	4
SI	1	4	-3	5
NL	2	-3	-1	6
MT	1	6	-5	7
PT	0	6	-6	8

How sensitive are the results (final rankings) to small perturbations in the weights?

- There is still *the need to define weights in pairwise comparison methods*... but with the advantage that no other methodological and conceptual choices are required (normalisation, outlier treatment, aggregation formula, etc.)
- But, how to summarise the impact of weights on the final ordering? → *Simulation approach*: generate scenarios with random sets of weights which depart from the equal weighting baseline scenario; sensitivity to weights can be measured in terms of variability of performance (support scores) in the resulting rankings

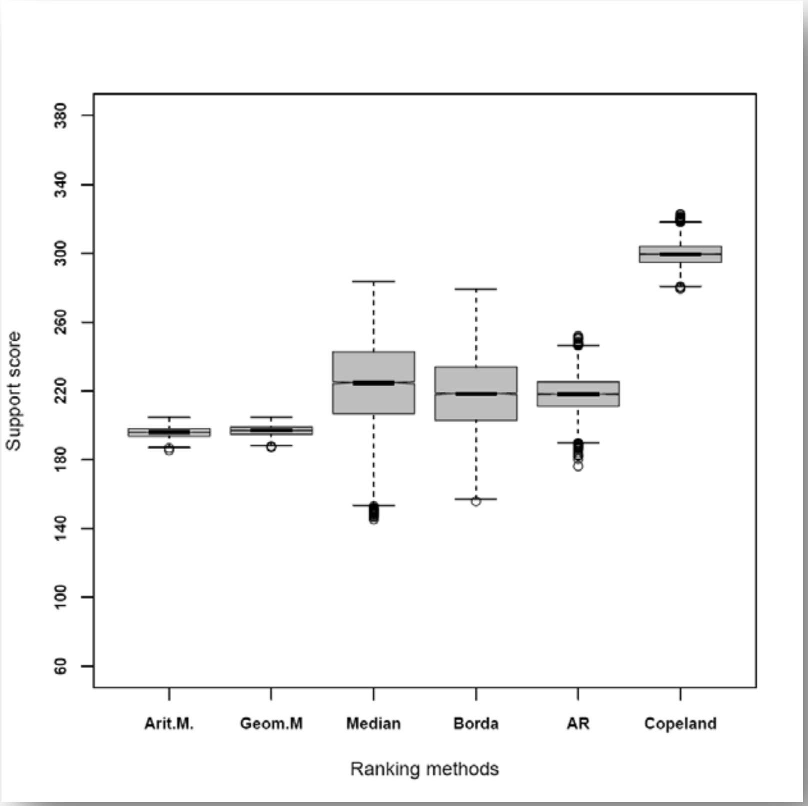
The Social Scoreboard of the European Pillar of Social Rights



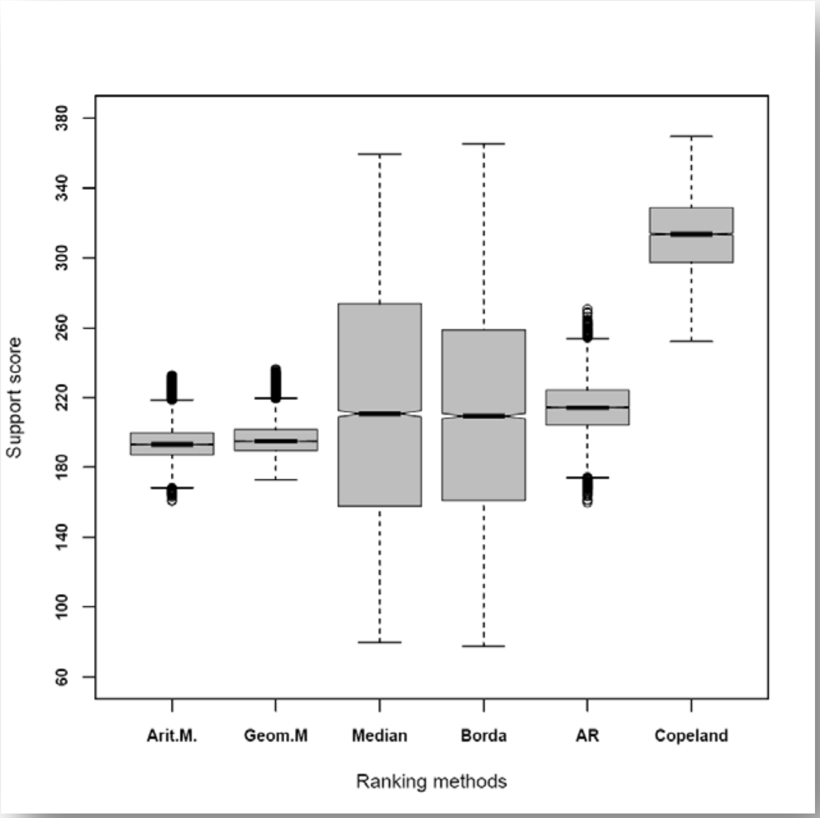
- Tracks trends and performances across EU in 3 broad themes and 12 policy areas
- Informs policy guidance in the context of the European Semester of economic policy coordination, assessing progress towards a social 'triple A'.
- 14 headline indicators singled out by the developers; a number of secondary indicators and additional breakdowns (per gender, age, educational attainment, etc.) reported in the expanded version

How sensitive are country rankings of social performance across different ordering methods to changes in the weights assigned to each criteria/headline indicator?

The Social Scoreboard of the European Pillar of Social Rights



Social Pillar 2015, random weights (10,000 rep), up to 10%



Social Pillar 2015, random weights (10,000 rep), up to 40%

- Weights are allowed to diverge from the 1/14 equal weights baseline scenario (up to 10% or 40% max for ind. indicators)
- ***Comparison*** of aggregation methods (*work in progress*): ***Copeland seems to perform consistently better*** in the context of small/moderate perturbations in weights

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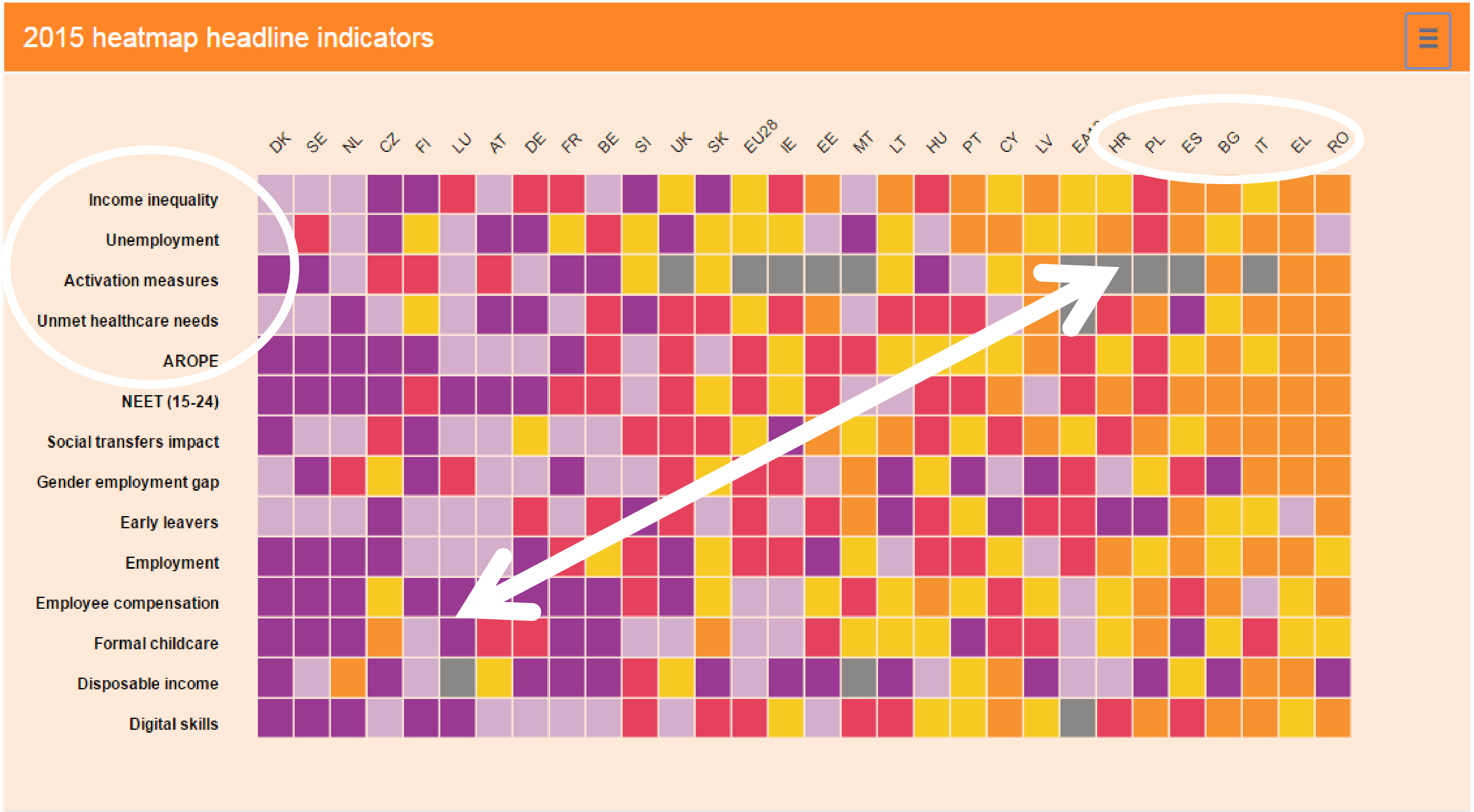
The Social Scoreboard of the European Pillar of Social Rights

[A little preview of the CoP] And lastly, how about ***a double re-ordering (based on the Copeland rule)*** of both countries and headline indicators?



__From a scoreboard to informed policy decisions__: The re-ordered heatmap of the 14 headline indicators of the Social Scoreboard ***reveals*** that the EU is facing most ***challenges*** on ***areas*** related to *income inequality*, *unemployment*, *activation measures*, *unmet healthcare needs*, and *AROPE*. And the ***countries*** that are most ***in need of further action*** are *Romania, Greece, Italy, Bulgaria, Spain, Poland and Croatia*.

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Suggested readings

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