The European Commission’s science and knowledge service

Joint Research Centre

European Commission

Step 6: Aggregation rules

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Outline on Aggregation rules

- Based on Scores
  1. Arithmetic Mean
  2. Geometric Mean
- Based on Ranks
  3. Median rank
  4. Majority
  5. Borda’s Count
- Based on Pairwise comparison
  6. Condorcet
  7. Kemeny
  8. Arrow – Raynaud
  9. Copeland

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Arithmetic mean

The ***arithmetic mean*** of a list of *n* real numbers equals:

$$\frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

This is the simplest, most obvious and most widespread aggregation method

Perfect substitutability – compensates bad performance in one aspect with good performance in another

Arithmetic mean, example

The score corresponding to the 4th pillar of the GTCI index in country *i* is calculated as The arithmetic average of sub-pillars 4.1 and 4.2:

Sustainability score = 37.04
Lifestyle score = 59.60

Retain pillar score = ½ (37.04+59.60) = 48.32

4	**RETAIN**	**48.32**	**70**
4.1	Sustainability	37.04	78
4.1.1	Pension system	37.37	56
4.1.2	Taxation	43.58	73
4.1.3	Brain retention	30.16	94
4.2	Lifestyle	59.60	64
4.2.1	Environmental performance	69.58	57
4.2.2	Personal safety	62.00	56
4.2.3	Physician density	14.55	78
4.2.4	Sanitation	92.27	57

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Multiplicative aggregations

The ***geometric mean*** of a list of *n* <u>*positive*</u> real numbers equals:

$$\sqrt[n]{\prod_{i=1}^{n} x_i} = \sqrt[n]{x_1 \times x_2 \times \cdots \times x_n}$$

Partial substitutability – compensates, but penalises low performance in some aspects

Geometric mean

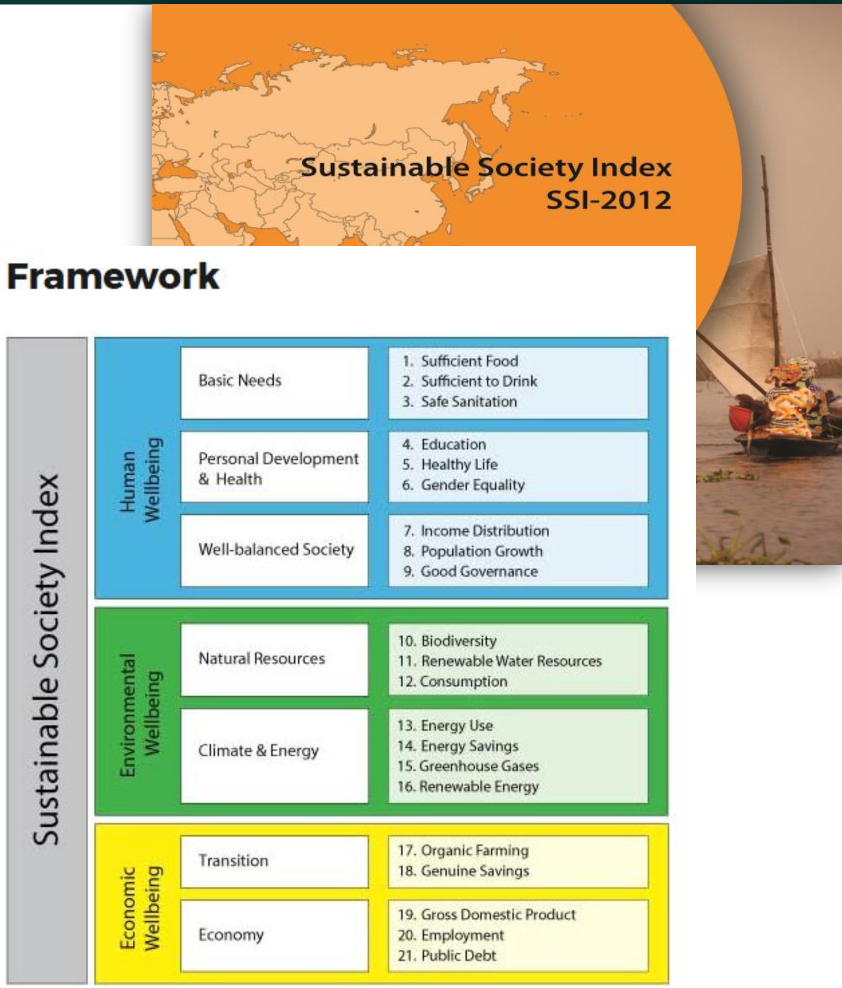
Basic Needs indicators - Country *i*	value
% people with sufficient food	92
% people with safe drinking water	79
% people with safe sanitation	17

$$\text{Sufficient Food} = \frac{92}{10} = 9.2$$

$$\text{Sufficient to Drink} = \frac{79}{10} = 7.9$$

$$\text{Safe Sanitation} = \frac{17}{10} = 1.7$$

$$\text{Basic Needs} = \sqrt[3]{9.2 \cdot 7.9 \cdot 1.7} = 4.98$$



Arithmetic vs Geometric mean: a ‘typical’ monkey’s dilemma ☺

Safe: 1
Tasty: 10

A.M.: **5.5**
G.M.: 3.2

Today the monkey thinks a big danger or a bad taste can not be compensated

The geometric mean is used

Safe: 5
Tasty: 5

A.M.: 5.0
G.M.: **5.0**

Safe: 9
Tasty: 2

A.M.: **5.5**
G.M.: 4.2

The balancing effect of geometric mean

	Sufficient Food	Sufficient to Drink	Safe Sanitation	Basic Needs (arithmetic)	Country *i's* improvement	Basic Needs (geometric)	Country *i's* improvement
Country *i* (t)	10.0	8.6	1.4	6.7		4.9	
Following year							
[a] Country *i* (t+1)	10.0	**9.6** **+1**	1.4	7.0	4.5%	5.1	4.1%
[b] Country *i* (t+1)	10.0	8.6	**2.4** **+1**	7.0	4.5%	5.9	20.4%
[c] Country *i* (t+1)	10.0	**9.6** **+1**	**0.4** **-1**	6.7	0%	3.4	-31.1%
[d] Country *i* (t+1)	10.0	**7.6** **-1**	**2.4** **+1**	6.7	0%	5.7	15.7%

1) *poor performance is* ***not completely compensated*** *by another good performance*;

2) ***rewards balance*** by ***penalizing*** low and unbalanced values;

3) ***encourages (policy) improvements in the weak dimensions***.

And what about weights?

- **Weighted linear aggregation**

For a sequence of positive weights w_i, with $\Sigma w_i=1$, the ***weighted arithmetic mean*** equals:

$$\sum_{i=1}^{n} w_i x_i \;=\; w_1 x_1 + w_2 x_2 + \cdots + w_n x_n$$

- **Weighted geometric aggregation**

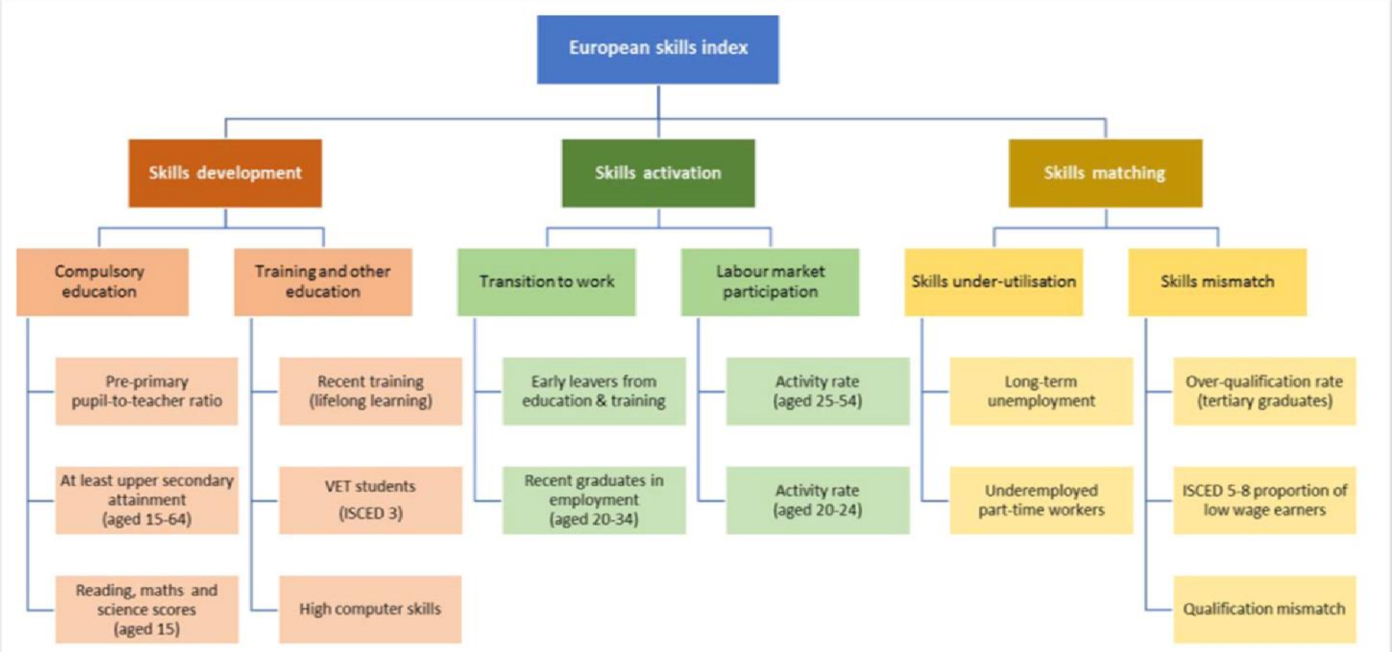
For a sequence of positive weights w_i, with $\Sigma w_i=1$, the ***weighted geometric mean*** equals:

$$\prod_{i=1}^{n} {x_i}^{w_i} \;= {x_1}^{w_1} \times {x_2}^{w_2} \times \cdots \times {x_n}^{w_n}$$

Hybrid aggregations

Mixed approach: to create CIs using more than one aggregation functions at different levels of aggregation

European Skills Index (CEDEFOP)	Arithmetic average within dimensions	Geometric average across dimensions



Source: European Skills Index (2018), Cedefop.

Similarities and Differences of means

Common features:

- Normalisation of indicators is required
- Extremely Sensitive to outliers (i.e. outlier treatment needed)
- Interval level information is kept in the output (scores, not ranks)
- Weights have the meaning of *trade-offs* (and not of importance coefficients): a lack in one dimension can be offset by a sufficient surplus in another

Differences:

- Perfect and constant substitutability (AM) vs. partial compensability (GM) (penalisation of unbalanced performances)
- Arithmetic means are always greater than or equal to equivalent geometric means

Aggregation with a mean

Before proceeding for a mean, check all the previous steps (normalisation, outliers, missing data, weights)

Remember: You need *quantitative* indicators

Quantitative and qualitative criteria together

Multi-criteria performance matrix
(quantitative/qualitative variables as constituent elements of the same conceptual framework)

	Criterion 1 (/20)	Criterion 2 (rating)	Criterion 3 (qual.)	Criterion 4 (Y/N)
Alternative 1	20	135	G	Yes
Alternative 2	9	156	B	Yes
Alternative 3	15	129	VG	No
Alternative 4	9	146	VB	No
Alternative 5	7	121	G	Yes
…	…	…	…	…

We need some method to compare, rank or evaluate the alternatives

From Social Choice Theory to Multi-Criteria Analysis

1	Ramon Llull
2	Nicolas de Condorcet
3	Nicholas of Kues
4	Jean-Charles, Chevalier de Borda

The problem of Social Choice:

- A group of voters has to select a candidate among a group of candidates (election)
- Each voter has a personal ranking of the candidates according to his/her preferences
- Which candidate must be elected?

Best interest of society

What is the «best» voting procedure?

Analogy with Multi-Criteria Analysis:

- Candidates ↔ Alternatives
- Voters ↔ Criteria

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Methods based on ranks

- Based on Scores
  1. Arithmetic Mean
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  3. Median rank
  4. Majority
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Methods based on ranks

Every criterion represents a voter, a point of view, and determines a complete ranking

	Criterion 1 (/20)	Criterion 2 (rating)	Criterion 3 (qual.)	Rank 1	Rank 2	Rank 3
Alternative 1	20	135	Good	1	3	2.5
Alternative 2	9	156	Bad	3.5	1	4
Alternative 3	15	129	Very Good	2	4	1
Alternative 4	9	146	Very Bad	3.5	2	5
Alternative 5	7	121	Good	5	5	2.5

Outline on Aggregation rules

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Median rank

<u>3 candidates:</u> **A**dam, **B**rian, **C**arlos

11 voters:

Rank candidates according to each voter (criteria) and then calculate median rank for each candidate <u>across voters</u>

5 voters	**4** voters	**2** voters
A	C	C
C	A	B
B	B	A

A: 11111**2**22233
B:22333**3**33333
C:11111**1**22222

Ranking
C
A
B

Outline on Aggregation rules

- Based on Scores
  1. Arithmetic Mean
  2. Geometric Mean
- Based on Ranks
  3. Median rank
  4. Majority (Relative Majority)
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Relative majority

Rank	Points
1	1
2	0
3	0
…	…
N-1	0
N	0

<u>3 candidates:</u> **A**dam, **B**rian, **C**arlos

30 voters, **Only the first position counts**

13 voters	**10** voters	**7** voters
A	B	C
B	C	B
C	A	A

17 voters out of 30 rank A as their least preferred option (strong opposition)

Problem: B and C preferred to A by a majority of voters!

A	13
B	10
C	7

Adam is elected

Outline on Aggregation rules

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Borda’s Count

Rank	Borda points
1	N-1
2	N-2
3	N-3
…	…
N-1	1
N	0

3 candidates: **A**dam, **B**rian, **C**arlos

Carlos = 39 x 2 + 31 x 1= 109

81 voters:

30 voters	**29** voters	**10** voters	**10** voters	**1** voter	**1** voter
A	C	C	B	A	B
C	A	B	A	B	C
B	B	A	C	C	A

Points
2
1
0

Scores	
A	**101**
B	**33**
C	**109**

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Borda's Count with indicators' weights

Rank	Borda points
1	N-1
2	N-2
3	N-3
…	…
N-1	1
N	0

Country **A,** Country **B,** Country **C**

Weights of 6 indicators instead of voters:

Country **C** = 0.45 x 2 + 0.25 x 1= 1.15

Ind. 1	Ind. 2	Ind. 3	Ind. 4	Ind. 5	Ind. 6
0.05 weight	**0.30** weight	**0.15** weight	**0.10** Weight	**0.10** weight	**0.20** weight
A	C	C	B	A	B
C	A	B	A	B	C
B	B	A	C	C	A

Points
2
1
0

Scores	
A	**0.70**
B	**0.85**
C	**1.15**

Borda’s count: irrelevant alternatives

Rank	Borda points
1	N-1
2	N-2
3	N-3
…	…
N-1	1
N	0

4 candidates: **A**dam, **B**rian, **C**arlos, **D**avid

7 voters:

3 voters	**2** voters	**2** voters	Points
C	B	A	3
B	A	D	2
A	D	C	1
D	C	B	0

Scores	
A	13
B	12
C	11
D	6

Ranking
A
B
C
D

Adam is elected

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Borda's count: irrelevant alternatives/2

C	B	A
B	A	~~D~~
A	~~D~~	C
~~D~~	C	B

Let's exclude the lowest

4 candidates: **A**dam, **B**rian, **C**arlos, ~~**D**avid~~

7 voters:

3 voters	**2** voters	**2** voters	Points
C	B	A	2
B	A	C	1
A	C	B	0

Just by dropping the last in the ranking, the order of preference for the better ranked alternatives changed

Problem: Borda's count is dependant on irrelevant alternatives (risk of manipulation)

Scores	
A	6
B	7
C	8

Ranking
C
B
A

Carlos is elected

Methods based on Outranking Matrix (OM)

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Methods based on pairwise comparisons

	Criterion 1 (/20)	Criterion 2 (rating)	Criterion 3 (qual.)	Criterion 4 (Y/N)	…
Alternative 1	20	135	G	Yes	…
Alternative 2	9	156	B	Yes	…
Alternative 3	15	129	VG	No	…
Alternative 4	9	146	VB	No	…
Alternative 5	7	121	G	Yes	…
…	…	…	…	…	…

Confront alternatives using the original values, all criteria simultaneously

Example: Alternative 1 is better than Alternative 5

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Condorcet's Method

3 candidates: **A**dam, **B**rian, **C**arlos

30 voters:

11 voters	**10** voters	**9** voters
A	B	C
B	C	B
C	A	A

Search for a **Condorcet winner**, i.e. an alternative preferred over every other in pairwise comparisons

OUTRANKING MATRIX

	A	B	C
A	0	11	11
B	19	0	21
C	19	9	0

Brian is elected

Limit: Condorcet's Cycle

3 candidates: **A**dam, **B**rian, **C**arlos

9 voters:

4 voters	**3** voters	**2** voters
A	B	C
B	C	A
C	A	B

Problem: The Condorcet winner might not exist! (cycle)

	A	B	C
A	0	6	4
B	3	0	7
C	5	2	0

A > B, B > C and C> A

None is elected

Quantitative data, weak structure

Problem: Data might be plagued with ***poor or*** even ***negative correlations*** among indicators. As well as voters, indicators may be independent and even opposite

Solution: Social Choice Theory is a valid alternative. It is meant for voters, they are not expected to be related and dependent.

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Quantitative data, weak structure

Avoid standard aggregation (*averaging*) methods, because results would then be ***highly sensitive*** to underlying methodological and conceptual choices done in the previous steps

- conceptual grouping of indicators into themes
- missing data estimation method (or none…)
- data treatment (of highly skewed variables)
- data normalisation
- weights
- aggregation formula

You would not be able to trust the results in such situation

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The ESRB Heatmap dataset, descriptives

The ESRB risk dashboard is a set of quantitative and qualitative indicators of systemic risk in the EU financial system

			average	*st.d.*	*min*	*max*	*skew*	*kurt*
MACRO	**Current real GDP growth**	**2.1**	0.10	2.60	-7.92	5.99	-0.12	1.07
MACRO	**Domestic credit-to-GDP gap**	**2.2**	-6.77	6.06	-21.88	0.41	-1.11	0.30
MACRO	**Current account balance-to-GDP ratio**	**2.3**	0.02	4.09	-9.89	10.27	0.65	0.18
MACRO	**Rate of unemployment**	**2.4**	10.83	5.17	4.15	26.22	1.35	1.83
FISCAL	**Forecast government debt-to-GDP ratio**	**2.5**	67.83	35.26	6.25	170.32	0.65	0.30
FISCAL	**Forecast government deficit-to-GDP ratio**	**2.6**	4.02	2.83	0.15	13.38	0.93	0.91
FISCAL	**Credit default swap premia on sovereign debt**	**2.7**	574.16	1836.22	18.63	12447.07	**5.57**	**31.71**
FISCAL	**Annual sovereign debt redemptions as a share of GDP**	**2.8**	14.51	11.07	0.00	47.37	0.81	-0.09
HH	**Households' debt-to-gross disposable income ratio**	**2.9**	104.67	61.58	36.88	268.92	1.23	0.86
HH	**Estimates of the over/undervaluation of residential property prices**	**3.1.a.**	2.58	11.85	-12.67	28.39	0.62	-0.93
HH	**Share of foreign currency loans on total loans to non-MFIs**	**3.2a**	18.81	25.58	0.28	89.45	1.55	0.97
HH	**MFI lending to HH (annual growth rates) NEW**	**n.a.2**	0.66	4.75	-16.69	11.12	-0.68	1.92
NFC	**Non-financial corporations' debt-to-GDP ratio**	**2.13**	115.88	74.87	0.00	555.04	**2.74**	**14.13**
NFC	**MFI lending to NFC (annual growth rates) NEW**	**n.a.1**	0.50	5.02	-10.79	14.01	0.23	-0.41
MFIs	**Share of central bank funding in credit institutions liabilities**	**4.5**	4.60	7.28	0.00	34.78	**2.58**	**7.24**
MFIs	**MFI's exposure to domestic sovereign (share of total assets) NEW**	**n.a.3**	0.08	0.06	0.00	0.23	0.92	-0.11
MFIs	**Banking sector leverage NEW**	**n.a.4**	16.16	7.22	4.98	50.46	1.35	**4.87**
MFIs	**Loan to deposit ratio NEW**	**n.a.5**	1.31	0.47	0.61	2.97	1.93	**4.43**

Notes: European Systemic Risk Board, raw data, pooled dataset: 2013Q3, 2012Q4, 2012 Q3, 2011 Q4 (four time-points x 28 countries)

Outliers in three indicators: problematic when analyzing the correlation structure

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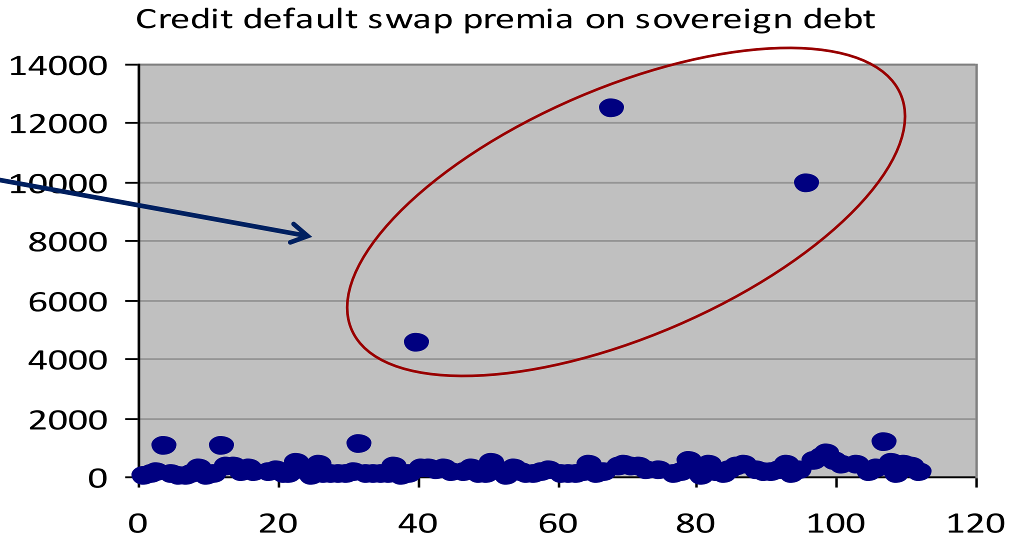
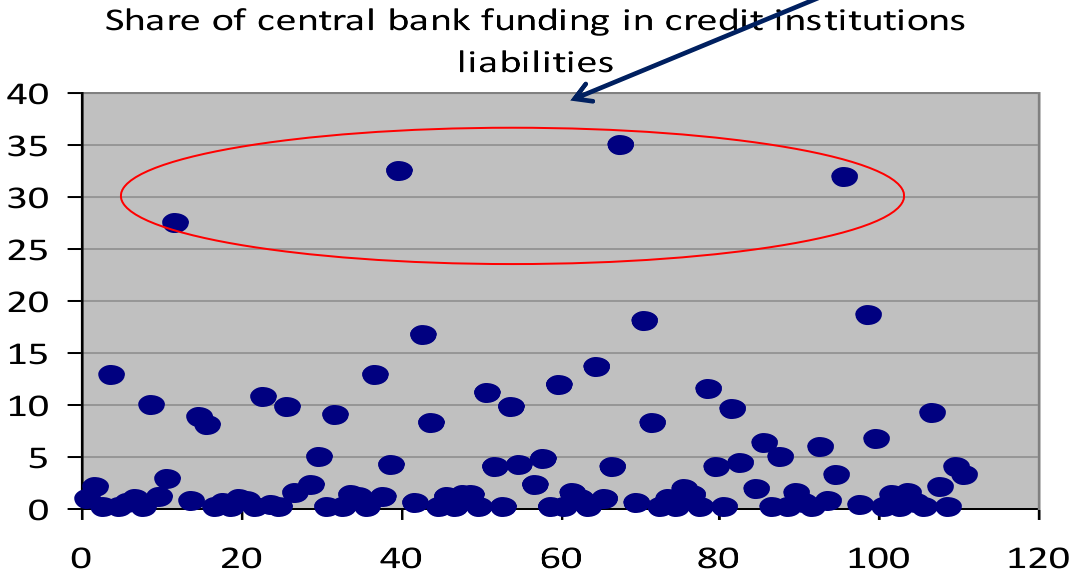
The ESRB Heatmap dataset

			average	st.d.	min	max	skew	kurt
MACRO	Current real GDP growth	2.1	0.10	2.60	-7.92	5.99	-0.12	1.07
MACRO	Domestic credit-to-GDP gap	2.2	-6.77	6.06	-21.88	0.41	-1.11	0.30
MACRO	Current account balance-to-GDP ratio	2.3	0.02	4.09	-9.89	10.27	0.65	0.18
MACRO	Rate of unemployment	2.4	10.83	5.17	4.15	26.22	1.35	1.83
FISCAL	Forecast government debt-to-GDP ratio	2.5	67.83	35.26	6.25	170.32	0.65	0.30
FISCAL	Forecast government deficit-to-GDP ratio	2.6	4.02	2.83	0.15	13.38	0.93	0.91
FISCAL	Credit default swap premia on sovereign debt	2.7	574.16	1836.22	18.63	12447.07	**5.57**	**31.71**
FISCAL	Annual sovereign debt redemptions as a share of GDP	2.8	14.51	11.07	0.00	47.37	0.81	-0.09
HH	Households' debt-to-gross disposable income ratio	2.9	104.67	61.58	36.88	268.92	1.23	0.86
HH	Estimates of the over/undervaluation of residential property prices	3.1.a.	2.58	11.85	-12.67	28.39	0.62	-0.93
HH	Share of foreign currency loans on total loans to non-MFIs	3.2a	18.81	25.58	0.28	89.45	1.55	0.97
HH	MFI lending to HH (annual growth rates) NEW	n.a.2	0.66	4.75	-16.69	11.12	-0.68	1.92
NFC	Non-financial corporations' debt-to-GDP ratio	2.13	115.88	74.87	0.00	555.04	**2.74**	**14.13**
NFC	MFI lending to NFC (annual growth rates) NEW	n.a.1	0.50	5.02	-10.79	14.01	0.23	-0.41
MFIs	Share of central bank funding in credit institutions liabilities	4.5	4.60	7.28	0.00	34.78	**2.58**	**7.24**
MFIs	MFI's exposure to domestic sovereign (share of total assets) NEW	n.a.3	0.08	0.06	0.00	0.23	0.92	-0.11
MFIs	Banking sector leverage NEW	n.a.4	16.16	7.22	4.98	50.46	1.35	**4.87**
MFIs	Loan to deposit ratio NEW	n.a.5	1.31	0.47	0.61	2.97	1.93	**4.43**





Need to be treated before the correlation analysis and calculation of an aggregate score based on linear/geometric aggregations

Shouldn't be treated: real extreme behaviours need to be accounted

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The ESRB Heatmap dataset

Correlation structure in the ESRB country heat map

	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
2.1	[1 ,1]	[-0.2 ,0.1]	[0.1 ,0.2]	[-0.4 ,-0.2]	[-0.7 ,-0.6]	[-0.6 ,-0.2]	[-0.7 ,-0.5]	[-0.6 ,-0.4]	[-0.4 ,-0.2]
2.2	[-0.2 ,0.1]	[1 ,1]	[-0.3 ,0]	[-0.6 ,-0.1]	[-0.2 ,0.1]	[-0.5 ,-0.3]	[-0.1 ,0]	[0 ,0.3]	[-0.5 ,-0.5]
2.3	[0.1 ,0.2]	[-0.3 ,0]	[1 ,1]	[-0.6 ,-0.2]	[-0.4 ,-0.2]	[-0.5 ,-0.3]	[-0.6 ,-0.4]	[-0.2 ,0]	[0.4 ,0.5]
2.4	[-0.4 ,-0.2]	[-0.6 ,-0.1]	[-0.6 ,-0.2]	[1 ,1]	[0.2 ,0.5]	[0.6 ,0.7]	[0.1 ,0.8]	[0 ,0.2]	[-0.1 ,0.1]
2.5	[-0.7 ,-0.6]	[-0.2 ,0.1]	[-0.4 ,-0.2]	[0.2 ,0.5]	[1 ,1]	[0.5 ,0.7]	[0.6 ,0.6]	[0.6 ,0.7]	[0.1 ,0.2]
2.6	[-0.6 ,-0.2]	[-0.5 ,-0.3]	[-0.5 ,-0.3]	[0.6 ,0.7]	[0.5 ,0.7]	[1 ,1]	[0.4 ,0.6]	[0.2 ,0.4]	[0.1 ,0.2]
2.7	[-0.7 ,-0.5]	[-0.1 ,0]	[-0.6 ,-0.4]	[0.1 ,0.8]	[0.6 ,0.6]	[0.4 ,0.6]	[1 ,1]	[0.2 ,0.6]	[-0.2 ,0.1]
2.8	[-0.6 ,-0.4]	[0 ,0.3]	[-0.2 ,0]	[0 ,0.2]	[0.6 ,0.7]	[0.2 ,0.4]	[0.2 ,0.6]	[1 ,1]	[-0.1 ,0.3]
2.9	[-0.4 ,-0.2]	[-0.5 ,-0.5]	[0.4 ,0.5]	[-0.1 ,0.1]	[0.1 ,0.2]	[0.1 ,0.2]	[-0.2 ,0.1]	[-0.1 ,0.3]	[1 ,1]
3.1.a.	[-0.3 ,0.3]	[0.1 ,0.5]	[-0.4 ,-0.3]	[-0.1 ,0.1]	[-0.5 ,0.2]	[-0.3 ,0.1]	[-0.4 ,0.2]	[-0.2 ,0.5]	[-0.5 ,-0.1]
3.2a	[0.4 ,0.6]	[-0.1 ,0]	[-0.3 ,-0.2]	[0 ,0.2]	[-0.4 ,-0.3]	[-0.3 ,0]	[-0.1 ,0.1]	[-0.4 ,-0.3]	[-0.4 ,-0.3]
n.a.2	[0 ,0.2]	[0.2 ,0.5]	[0.1 ,0.1]	[-0.4 ,-0.1]	[-0.4 ,-0.3]	[-0.3 ,-0.2]	[-0.5 ,-0.3]	[-0.3 ,0]	[-0.1 ,0.1]
2.13	[-0.3 ,-0.1]	[-0.5 ,-0.4]	[0.2 ,0.4]	[-0.1 ,0.1]	[0 ,0.1]	[-0.1 ,0.1]	[0 ,0.2]	[0.1 ,0.2]	[0.5 ,0.5]
n.a.1	[0 ,0.4]	[0.5 ,0.6]	[-0.1 ,-0.1]	[-0.4 ,-0.3]	[-0.5 ,-0.2]	[-0.5 ,-0.2]	[-0.6 ,-0.3]	[-0.2 ,0.1]	[-0.2 ,0]
4.5	[-0.7 ,-0.5]	[-0.5 ,-0.1]	[-0.4 ,-0.2]	[0.3 ,0.7]	[0.8 ,0.8]	[0.6 ,0.8]	[0.7 ,0.8]	[0.4 ,0.4]	[0.1 ,0.2]
n.a.3	[-0.2 ,-0.1]	[0.3 ,0.6]	[-0.5 ,-0.2]	[0.1 ,0.3]	[0 ,0.1]	[0.1 ,0.2]	[0 ,0.2]	[0 ,0.3]	[-0.6 ,-0.5]
n.a.4	[-0.6 ,-0.4]	[-0.2 ,-0.1]	[0.1 ,0.4]	[-0.2 ,0.3]	[0.4 ,0.6]	[-0.1 ,0.4]	[-0.2 ,0.5]	[0.2 ,0.6]	[0.4 ,0.6]
n.a.5	[-0.2 ,-0.1]	[-0.4 ,-0.3]	[0.3 ,0.4]	[0 ,0.1]	[-0.1 ,-0.1]	[-0.1 ,0.1]	[-0.2 ,-0.1]	[-0.2 ,-0.1]	[0.5 ,0.6]

	3.1.a.	3.2a	n.a.2	2.13	n.a.1	4.5	n.a.3	n.a.4	n.a.5
2.1	[-0.3 ,0.3]	[0.4 ,0.6]	[0 ,0.2]	[-0.3 ,-0.1]	[0 ,0.4]	[-0.7 ,-0.5]	[-0.2 ,-0.1]	[-0.6 ,-0.4]	[-0.2 ,-0.1]
2.2	[0.1 ,0.5]	[-0.1 ,0]	[0.2 ,0.5]	[-0.5 ,-0.4]	[0.5 ,0.6]	[-0.5 ,-0.1]	[0.3 ,0.6]	[-0.2 ,-0.1]	[-0.4 ,-0.3]
2.3	[-0.4 ,-0.3]	[-0.3 ,-0.2]	[0.1 ,0.1]	[0.2 ,0.4]	[-0.1 ,-0.1]	[-0.4 ,-0.2]	[-0.5 ,-0.2]	[0.1 ,0.4]	[0.3 ,0.4]
2.4	[-0.1 ,0.1]	[0 ,0.2]	[-0.4 ,-0.1]	[-0.1 ,0.1]	[-0.4 ,-0.3]	[0.3 ,0.7]	[0.1 ,0.3]	[-0.2 ,0.3]	[0 ,0.1]
2.5	[-0.5 ,0.2]	[-0.4 ,-0.3]	[-0.4 ,-0.3]	[0 ,0.1]	[-0.5 ,-0.2]	[0.8 ,0.8]	[0 ,0.1]	[0.4 ,0.6]	[-0.1 ,-0.1]
2.6	[-0.3 ,0.1]	[-0.3 ,0]	[-0.3 ,-0.2]	[-0.1 ,0.1]	[-0.5 ,-0.2]	[0.6 ,0.8]	[0.1 ,0.2]	[-0.1 ,0.4]	[-0.1 ,0.1]
2.7	[-0.4 ,0.2]	[-0.1 ,0.1]	[-0.5 ,-0.3]	[0 ,0.2]	[-0.6 ,-0.3]	[0.7 ,0.8]	[0 ,0.2]	[-0.2 ,0.5]	[-0.2 ,-0.1]
2.8	[-0.2 ,0.5]	[-0.4 ,-0.3]	[-0.3 ,0]	[0.1 ,0.2]	[-0.2 ,0.1]	[0.4 ,0.4]	[0 ,0.3]	[0.2 ,0.6]	[-0.2 ,-0.1]
2.9	[-0.5 ,-0.1]	[-0.4 ,-0.3]	[-0.1 ,0.1]	[0.5 ,0.5]	[-0.2 ,0]	[0.1 ,0.2]	[-0.6 ,-0.5]	[0.4 ,0.6]	[0.5 ,0.6]
3.1.a.	[1 ,1]	[-0.4 ,0]	[0 ,0.7]	[0 ,0.3]	[0.2 ,0.4]	[-0.4 ,0.3]	[-0.1 ,0.1]	[0.2 ,0.4]	[-0.2 ,0.3]
3.2a	[-0.4 ,0]	[1 ,1]	[-0.4 ,-0.4]	[-0.4 ,-0.3]	[0 ,0.3]	[-0.3 ,-0.3]	[0.1 ,0.1]	[-0.5 ,-0.4]	[-0.2 ,-0.1]
n.a.2	[0 ,0.7]	[-0.4 ,-0.4]	[1 ,1]	[-0.1 ,0.1]	[0.2 ,0.6]	[-0.5 ,-0.2]	[0 ,0.1]	[-0.1 ,0]	[-0.1 ,0]
2.13	[0 ,0.3]	[-0.4 ,-0.3]	[-0.1 ,0.1]	[1 ,1]	[-0.4 ,-0.2]	[0.1 ,0.2]	[-0.6 ,-0.6]	[0.1 ,0.4]	[0.1 ,0.2]
n.a.1	[0.2 ,0.4]	[0 ,0.3]	[0.2 ,0.6]	[-0.4 ,-0.2]	[1 ,1]	[-0.6 ,-0.3]	[0 ,0.4]	[-0.3 ,0.1]	[-0.3 ,0]
4.5	[-0.4 ,0.3]	[-0.3 ,-0.3]	[-0.5 ,-0.2]	[0.1 ,0.2]	[-0.6 ,-0.3]	[1 ,1]	[0 ,0.1]	[0.2 ,0.5]	[0 ,0.1]
n.a.3	[-0.1 ,0.1]	[0.1 ,0.1]	[0 ,0.1]	[-0.6 ,-0.6]	[0 ,0.4]	[0 ,0.1]	[1 ,1]	[-0.4 ,-0.3]	[-0.3 ,-0.3]
n.a.4	[0.2 ,0.4]	[-0.5 ,-0.4]	[-0.1 ,0]	[0.1 ,0.4]	[-0.3 ,0.1]	[0.2 ,0.5]	[-0.4 ,-0.3]	[1 ,1]	[0.2 ,0.4]
n.a.5	[-0.2 ,0.3]	[-0.2 ,-0.1]	[-0.1 ,0]	[0.1 ,0.2]	[-0.3 ,0]	[0 ,0.1]	[-0.3 ,-0.3]	[0.2 ,0.4]	[1 ,1]

Notes: raw data (without outliers), pooled dataset: 2013Q3, 2012Q4, 2012 Q3, 2011 Q4, correlations less than 0.38 are not significant

Weak correlation structure: poor (or negative) correlations / correlation structure changing over time

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The ESRB Heatmap dataset

Correlation structure in the ESRB country heat map

Example: Macro dimension

	2.1	2.2	2.3	2.4
2.1	[1,1]	[-0.2,0.1]	[0.1,0.2]	[-0.4,-0.2]
2.2	[-0.2,0.1]	[1,1]	[-0.3,0]	[-0.6,-0.1]
2.3	[0.1,0.2]	[-0.3,0]	[1,1]	[-0.6,-0.2]
2.4	[-0.4,-0.2]	[-0.6,-0.1]	[-0.6,-0.2]	[1,1]

Notes: raw data (without outliers), pooled dataset: 2013Q3, 2012Q4, 2012 Q3, 2011 Q4, correlations less than 0.38 are not significant

- Most bivariate correlations are not significant at any of the 4 time-points
- No bivariate correlation is significant at all four time points
- Presence of significantly negative correlations

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The outranking matrix
(A matrix of pairwise comparisons)

For every pair of alternatives/countries, check which one is the winner of a direct comparison (considering the weights)

Indicator	I.1	I.2	I.3
Weights	0.35	0.45	0.2
A	3	Very Bad	205
B	4	Bad	48
C	3	Very Bad	88
D	6	Very Good	446
E	2	Good	208
F	5	Very Bad	18
G	3	Good	351
H	5	Bad	88

Example 1:

A versus B = 0.20

B versus A = 0.80

Example 2:

D versus G = 1.00

G versus D = 0.00

Outranking Matrix – Construction

Data 2013	Fiscal Dimension			
	2.5	2.6	2.7	2.8
Orientation	-1	-1	-1	-1
Weights	0.25	0.25	0.25	0.25
MT	74.9	3.6	205.0	14.7
NL	75.8	3.5	47.9	11.6
PL	58.9	4.1	87.9	5.8
PT	124.3	4.0	445.5	18.8
RO	38.5	2.4	207.6	7.3
SE	39.0	0.4	17.7	7.0
SI	66.5	4.9	350.9	5.6
SK	56.7	3.1	87.8	9.2

Step 1 – Raw data, Weights & Orientation

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Outranking Matrix – Construction

Data 2013	Fiscal Dimension			
	2.5	2.6	2.7	2.8
Orientation	-1	-1	-1	-1
Weights	0.25	0.25	0.25	0.25
MT	74.9	3.6	205.0	14.7
NL	75.8	3.5	47.9	11.6
PL	58.9	4.1	87.9	5.8
PT	124.3	4.0	445.5	18.8
RO	38.5	2.4	207.6	7.3
SE	39.0	0.4	17.7	7.0
SI	66.5	4.9	350.9	5.6
SK	56.7	3.1	87.8	9.2

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

For each pairwise comparison, the weights for the indicators that favour A versus B are added up = concordance index. In case of ties, weights are split between countries. **For *n* countries, there are *n (n-1)* pairwise comparisons to be made**

Example:
MT versus NL = 0.25
NL versus MT = 0.75

Sum = 1.00

Outranking Matrix – Construction

Data 2013	Fiscal Dimension			
	2.5	2.6	2.7	2.8
Orientation	-1	-1	-1	-1
Weights	0.25	0.25	0.25	0.25
MT	74.9	3.6	205.0	14.7
NL	75.8	3.5	47.9	11.6
PL	58.9	4.1	87.9	5.8
PT	124.3	4.0	445.5	18.8
RO	38.5	2.4	207.6	7.3
SE	39.0	0.4	17.7	7.0
SI	66.5	4.9	350.9	5.6
SK	56.7	3.1	87.8	9.2

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

For each comparison, the weights for the indicators that favour A versus B are added up = ***concordance index***. In case of ties, weights are split between countries.

For *n* countries, there are *n (n-1)* pairwise comparisons to be made

Example:
MT versus PT = 1.00
PT versus MT = 0.00

Sum = 1.00

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Outranking Matrix – Construction

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

Step 3 – Outranking matrix

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

All concordance values are entered in the outranking matrix.
(entries above and below the diagonal sum up to 1.0)

MT versus NL = 0.25
NL versus MT = 0.75

MT versus PT = 1.00
PT versus MT = 0.00

Outline on Aggregation rules

- Based on Scores

  1. Arithmetic Mean
  2. Geometric Mean

- Based on Ranks

  3. Median rank
  4. Majority (Relative Majority)
  5. Borda’s Count

- Based on Pairwise comparison

  6. Condorcet
  7. Kemeny (C-K-Y-L)
  8. Arrow – Raynaud
  9. Copeland

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Kemeny order: Condorcet-Kemeny-Young-Levenglick (C-K-Y-L)

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

Step 3 – Outranking matrix

Step 4 – Maximum Likelihood ranking (highest support score)

Find the **permutation of rankings** which **maximises the sum** of elements **above the diagonal**

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Kemeny order – Maximises Likelihood

	SE	RO	PL	SK	SI	NL	MT	PT
SE	0.00	**0.75**	**0.75**	**1.00**	**0.75**	**1.00**	**1.00**	**1.00**
RO	0.25	0.00	**0.50**	**0.75**	**0.75**	**0.75**	**0.75**	**1.00**
PL	0.25	0.50	0.00	**0.50**	**0.75**	**0.50**	**0.75**	**0.75**
SK	0.00	0.25	0.50	0.00	**0.75**	**0.75**	**1.00**	**1.00**
SI	0.25	0.25	0.25	0.25	0.00	**0.50**	**0.50**	**0.75**
NL	0.00	0.25	0.50	0.25	0.50	0.00	**0.75**	**1.00**
MT	0.00	0.25	0.25	0.00	0.50	0.25	0.00	**1.00**
PT	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

Step 3 – Outranking matrix

Step 4 – Maximum Likelihood ranking (highest support score)

Find the **permutation of rankings** which **maximises the sum** of elements **above the diagonal**

Kemeny order – Maximises Likelihood

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

Step 3 – Outranking matrix

Step 4 – Maximum Likelihood ranking (highest support score)

	SE	RO	PL	SK	SI	NL	MT	PT
SE	0.00	**0.75**	**0.75**	**1.00**	**0.75**	**1.00**	**1.00**	**1.00**
RO	0.25	0.00	**0.50**	**0.75**	**0.75**	**0.75**	**0.75**	**1.00**
PL	0.25	0.50	0.00	**0.50**	**0.75**	**0.50**	**0.75**	**0.75**
SK	0.00	0.25	0.50	0.00	**0.75**	**0.75**	**1.00**	**1.00**
SI	0.25	0.25	0.25	0.25	0.00	**0.50**	**0.50**	**0.75**
NL	0.00	0.25	0.50	0.25	0.50	0.00	**0.75**	**1.00**
MT	0.00	0.25	0.25	0.00	0.50	0.25	0.00	**1.00**
PT	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00

	Rank
SE	1
RO	2
PL	3
SK	4
SI	5
NL	6
MT	7
PT	8

A Kemeny order is not always unique!

Summary on Kemeny order

- Fully **non-compensatory** approach;
- no impact of outliers;
- no need for data normalisation
- no need for ”good” correlation structure;
- can be used both with continuous and categorical variables;

- ***only weights and orientation*** are ***required***;
- ***weights*** represent <u>exactly</u> ***the importance*** *of the indicator*;

- It is computationally **more** complicated then the ‘means’
- Software under development (JRC-COIN) within Excel, R, Matlab

Sources: Athanasoglou (2015) , Tarjan (1972), Van Zuylen, and Williamson (2009), Munda and Nardo (2009)

Outline on Aggregation rules

- Based on Scores
  1. Arithmetic Mean
  2. Geometric Mean
- Based on Ranks
  3. Median rank
  4. Majority (Relative Majority)
  5. Borda's Count
- Based on Pairwise comparison
  6. Condorcet
  7. Kemeny (C-K-Y-L)
  8. Arrow – Raynaud
  9. Copeland

Quick-searching algorithms have been developed to approximate the optimal solution of Kemeny order

Arrow-Raynaud algorithm

a) **Identify** max value along each row;
b) **Find** the row with minimum of the maxima;
c) **Delete** row and column of the row-country
(The rank of the country is the lowest position available);
d) **Repeat** step till the outranking matrix becomes void.

Step 1 – Raw data & Weights & Orientation

Step 2 – Concordance index

Step 3 – Outranking matrix

Step 4 – Arrow-Raynaud algorithm (Kemeny approximation)

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

Min of max

Arrow-Raynaud algorithm

PT shows the smallest value, among the rows maxima, it gets rank 8

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

	Rank
MT	
NL	
PL	
PT	**8**
RO	
SE	
SI	
SK	

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Arrow-Raynaud algorithm

	MT	NL	PL	RO	SE	SI	SK
MT	0.00	0.25	0.25	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.50	0.25	0.75	0.50
RO	0.75	0.75	0.50	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	0.25	0.00	0.75	0.00

Min of max

Alternative
Min of max

	Rank
MT	**7**
NL	
PL	
PT	**8**
RO	
SE	
SI	**7**
SK	

Arrow-Raynaud algorithm

	NL	PL	RO	SE	SI	SK
NL	0.00	0.50	0.25	0.00	0.50	0.25
PL	0.50	0.00	0.50	0.25	0.75	0.50
RO	0.75	0.50	0.00	0.25	0.75	0.75
SE	1.00	0.75	0.75	0.00	0.75	1.00
SI	0.50	0.25	0.25	0.25	0.00	0.25
SK	0.75	0.50	0.25	0.00	0.75	0.00

Min of max

	Rank
MT	**7**
NL	**6**
PL	
PT	**8**
RO	
SE	
SI	
SK	

Arrow-Raynaud algorithm

	PL	RO	SE	SI	SK
PL	0.00	0.50	0.25	0.75	0.50
RO	0.50	0.00	0.25	0.75	0.75
SE	0.75	0.75	0.00	0.75	1.00
SI	0.25	0.25	0.25	0.00	0.25
SK	0.50	0.25	0.00	0.75	0.00

Min of max

	Rank
MT	**7**
NL	**6**
PL	
PT	**8**
RO	
SE	
SI	**5**
SK	

Arrow-Raynaud algorithm

	PL	RO	SE	SK
PL	0.00	0.50	0.25	0.50
RO	0.50	0.00	0.25	0.75
SE	0.75	0.75	0.00	1.00
SK	0.50	0.25	0.00	0.00

Min of max

	Rank
MT	**7**
NL	**6**
PL	
PT	**8**
RO	
SE	
SI	**5**
SK	**4**

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Arrow-Raynaud algorithm

	PL	RO	SE
PL	0.00	0.50	0.25
RO	0.50	0.00	0.25
SE	0.75	0.75	0.00

Min of max

	Rank
MT	7
NL	6
PL	3
PT	8
RO	
SE	
SI	5
SK	4

Arrow-Raynaud algorithm

	RO	SE
RO	0.00	0.25
SE	0.75	0.00

Min of max

	Rank
MT	7
NL	6
PL	3
PT	8
RO	2
SE	1
SI	5
SK	4

Arrow-Raynaud algorithm

Ordered Outranking matrix

	SE	RO	PL	SK	SI	NL	MT	PT
SE	0.00	**0.75**	**0.75**	**1.00**	**0.75**	**1.00**	**1.00**	**1.00**
RO	0.25	0.00	**0.50**	**0.75**	**0.75**	**0.75**	**0.75**	**1.00**
PL	0.25	0.50	0.00	**0.50**	**0.75**	**0.50**	**0.75**	**0.75**
SK	0.00	0.25	0.50	0.00	**0.75**	**0.75**	**1.00**	**1.00**
SI	0.25	0.25	0.25	0.25	0.00	**0.50**	**0.50**	**0.75**
NL	0.00	0.25	0.50	0.25	0.50	0.00	**0.75**	**1.00**
MT	0.00	0.25	0.25	0.00	0.50	0.25	0.00	**1.00**
PT	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00

	Rank
MT	**7**
NL	**6**
PL	**3**
PT	**8**
RO	**2**
SE	**1**
SI	**5**
SK	**4**

Arrow-Raynaud algorithm

- Arrow-Raynaud algorithm selects rankings that resolve Condorcet cycles in a way that minimizes the maximum pairwise disagreement across all pairs of alternatives. The set of such rankings can be very LARGE.

11 Equivalent rankings resulting from our example

MT	**7**	7	7	7	7	7	7	6	6	6	6
NL	**6**	6	6	5	5	5	4	5	5	4	5
PL	**3**	4	2	4	3	2	5	4	3	5	2
PT	**8**	8	8	8	8	8	8	8	8	8	8
RO	**2**	2	3	2	2	3	2	2	2	2	3
SE	**1**	1	1	1	1	1	1	1	1	1	1
SI	**5**	5	5	6	6	6	6	7	7	7	7
SK	**4**	3	4	3	4	4	3	3	4	3	4

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Outline on Aggregation rules

- Based on Scores

    1. Arithmetic Mean
    2. Geometric Mean

- Based on Ranks

    3. Median rank
    4. Majority (Relative Majority)
    5. Borda’s Count

- Based on Pairwise comparison

    6. Condorcet
    7. Kemeny (C-K-Y-L)
    8. Arrow – Raynaud
    9. Copeland

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Copeland rule

How does Copeland work? ***Wins*** minus ***Defeats***

Outranking matrix

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

	Wins	Defeats	Total	Rank
SE	7	0	7	1
RO				
PL				
SK				
SI				
NL				
MT				
PT				

Copeland rule

If Kemeny is the optimal solution, Copeland is a good approximation with the advantage of "button-click" speed

Outranking matrix

	MT	NL	PL	PT	RO	SE	SI	SK
MT	0.00	0.25	0.25	1.00	0.25	0.00	0.50	0.00
NL	0.75	0.00	0.50	1.00	0.25	0.00	0.50	0.25
PL	0.75	0.50	0.00	0.75	0.50	0.25	0.75	0.50
PT	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00
RO	0.75	0.75	0.50	1.00	0.00	0.25	0.75	0.75
SE	1.00	1.00	0.75	1.00	0.75	0.00	0.75	1.00
SI	0.50	0.50	0.25	0.75	0.25	0.25	0.00	0.25
SK	1.00	0.75	0.50	1.00	0.25	0.00	0.75	0.00

	Wins	Defeats	Total	Rank
SE	7	0	7	1
RO	5	1	4	2
PL	3	1	2	3
SK	4	2	2	4
SI	1	4	-3	5
NL	2	-3	-1	6
MT	1	6	-5	7
PT	0	6	-6	8

The compensation dilemma

- Based on Scores
  1. Arithmetic Mean
  2. Geometric Mean
- Based on Ranks
  3. Median rank
  4. Majority (Relative Majority)
  5. Borda's Count

Compensatory approaches/ Aggregation of data

- Based on Pairwise comparison
  6. Condorcet
  7. Kemeny (C-K-Y-L)
  8. Arrow – Raynaud
  9. Copeland

Non-Compensatory approaches/ Comparison of alternatives

Suggested readings

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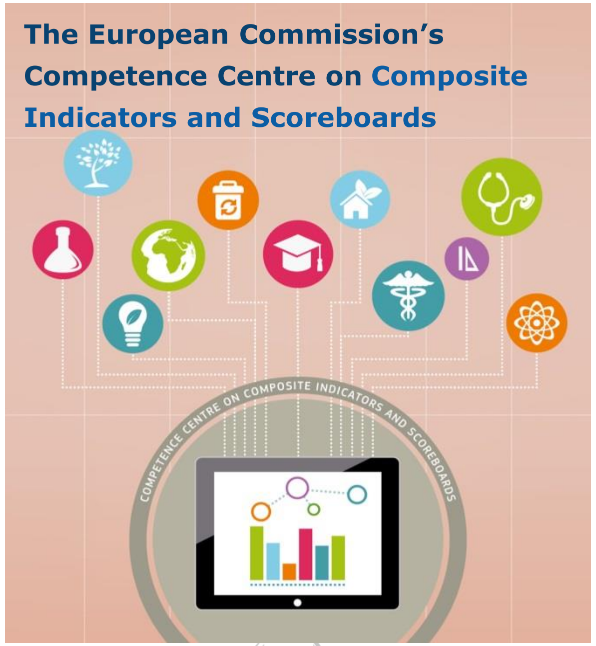

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